Exponential Functions: Monsters and Amoebas

Here's the situation:

Your spaceship has crashed on an unknown planet. You and your crew encounter a drooling, carnivorous alien monster. As you can guess, this is not good. It gets worse. While you are cowering in a cave, trying not to cry "mommy" in front of your crew, your science officer is able to chart the monster's growth over several hours' time. She comes back to your with her report (and minus one arm). The news is grim. With each hour that passes, the monster doubles in size (specifically, his height). The science officer also said that the monster's stomach was making those growly-hungry noises.

If we assume the monster is 1 foot tall at birth, what formula would describe the growth of the monster?

Well, let's figure it out. The best thing to do to figure out things like this is to make a chart of data. You can usually find a pattern that will lead to the formula.

TIME (t)	HEIGHT IN FEET
0	1
1	2
2	4
3	8
4	16
5	32
6	64

We can write **HEIGHT = 2^t** **2 is our common ratio (because it's doubling)**

Using function notation, our official formula is **$h(t) = 2^t$**

h for height **t is our input variable**

Now, you can figure out the monster's height at any time.

How tall will one of these monsters be 6 hours and 30 minutes after it's born? Let's find out.
The time (t) is 6.5, so we'll substitute that number into our formula:

$t = 6.5$
$h(t) = 2^t$
$h(6.5) = 2^{6.5} \approx 90.5$

After 6.5 hours, the monster will be about 90.5 feet tall. I'm sure you can only imagine the amount of drool something that big would produce . . . and how hungry it would be!

YOUR TURN

How tall would the monster be 4.7 hours after birth? $y = 2^{4.7} = 26$ feet

How tall would the monster be 3.2 hours after birth? $y = 2^{3.2} = 9.19$ feet

By the way, the function **h(t) = 2^t** is called an "**exponential function**", since the variable is **up** in the exponent. The 2 is called the base of the exponential function.

Now, back to the story . . .

For some reason, your science officer starts yelling crazy things at you (while waving around her one arm) and quits. In desperation, you promote the crew cook, Stu, and send him out of the cave to do more exploring. Unfortunately, he finds a second species of drooling alien monster. Fortunately for Stu, it's an herbivore! Unfortunately for Stu, your uniforms are green. Days later, one of Stu's socks and clip board of growth data for the monster is found.

Here's what Stu had written:

SECOND ALIEN SPECIES IS 4 INCHES TALL AT BIRTH WITH EACH HOUR THAT PASSES, THE MONSTER TRIPLES IN HEIGHT AND MAY BE TASTY BROILED WITH ONIONS.

Most of Stu's data had been covered in drool and made illegible. Based on his conclusion, complete the table for this species.

TIME (t)	HEIGHT IN INCHES	HEIGHT IN FEET
0	4	1/3
1	12	1
2	36	3
3	108	9
4	324	27
5	972	81
6	2916	243

Initial value common ratio

FORMULA FOR GROWTH: $h(t) = 4 \bullet 3^t$

YOUR TURN : Record your answers for the next two questions in inches and feet.

How tall with this species of monster be after 2.5 hours?

$y = 4(3)^{2.5} = 62.4$ inches or 5.2 feet

How tall will it be after 4.2 hours?

$y = 4(3)^{4.2} = 403.6$ inches or 33.6 feet

Back to the story . . .

You are sent out of the cave next. You find a third species of alien monster. It is not drooling, but it does have a serious breath problem. You find that this species is 2 feet tall at birth and gets 50% taller with each day that passes.

Using percentages is something you've been doing for ages – but when you're talking about growing 50% taller than you were before a little thinking is involved. We know that 50% is half of the original height...so that's 1 foot of growth. That means that the monster would be 3 feet tall the next day.

Complete the table and then use it find the common ratio.

TIME (t)	HEIGHT IN FEET
0	2
1	3
2	4.5
3	6.75
4	10.125
5	15.1875
6	22.78125

What is the common ratio for this monster's growth pattern?

$\frac{3}{2} = 1.5$

If you think about it, this makes sense! The monster keeps 100% of its original height and adds on 50% more. That's 150% total. When you change 150% to a decimal so you can use it you get 1.5...our growth factor!

Create a formula to describe the monster's growth. $h(t) = initial\ value \cdot common\ ratio^{time}$

$h(t) = 2(1.5)^t$

How tall will the monster be after 3.7 days?

$h(3.7) = 2(1.5)^{3.7} = 8.97$ feet

YOUR TURN

A twelve foot tall monster appears be 30% taller every hour! How tall will it be in twelve hours?

$h(t) = 12(1.3)^t$

$h(12) = 12(1.3)^{12} = \boxed{279.6 \text{ feet}}$

A 175 centimeter tall alien grows 22% taller every 30 minutes. How tall will it be in 5 hours? Write your answer in meters.

300 minutes

$300 \div 30 = 10$

$h(t) = 175(1.22)^t$

$h(10) = 175(1.22)^{10} = 1278.3$ cm

$1278.3 \div 100 = \boxed{12.78 \text{ m}}$

Going from large to small, you find another type of critter on the planet: alien amoebas! Since you're stranded on the planet and there's nothing else to do except run screaming from some of the inhabitants, you decide to study amoeba populations.

Typically animal populations grow by mommy and daddy animals making baby animals. (Hey – this lesson is G-rated!) Amoeba populations, on the other hand, grow by each amoeba splitting into two amoebas. So, if each alien amoeba splits into 2 amoebas every hour, can we create a formula to describe the growth of the amoeba population?

Let's say we start with 50 amoebas . . .

TIME (t)	NUMBER OF AMEOBAS
0	50
1	100
2	200
3	400
4	800
5	1600
6	3200

Look familiar?

After t hours: $50 \bullet 2^t$

Initial population: 50
Growth factor (in this case, how many times it splits in 1 hour): 2
Number of splits: t

Hey! This is just like alien monster growth!!

Okay, so, what if it gets more complicated?

If we start with 200 alien amoebas and each amoeba splits into 4 amoebas every 12 hours, how many amoebas will there be in a week?

Set it up and think!

Initial population: 200
Growth factor: 4
Number of splits: ?

Our split factor is for 12 hours . . . there are two 12 hours time periods in 1 day . . . and 7 days in a week . . . You're correct . . . the number of splits is 14.

So the equation looks like:

$$\text{Number of amoebas} = \text{initial population} \bullet \text{Growth factor}^{\text{time}}$$
$$= 200 \bullet 4^{14}$$

YOUR TURN

If we start with 30 alien amoebas and each amoeba splits into 15 amoebas every 10 minutes, how many amoebas will there be in an hour and a half? Write the equation first and then your solution.

$y = 30(15)^t \qquad t = \frac{90}{10} = 9$ splits

$y = 30(15)^9$

$= 1.15 \times 10^{12}$

$= 1,153,300,781,000$

Here's how I'd like for you to remember this formula:

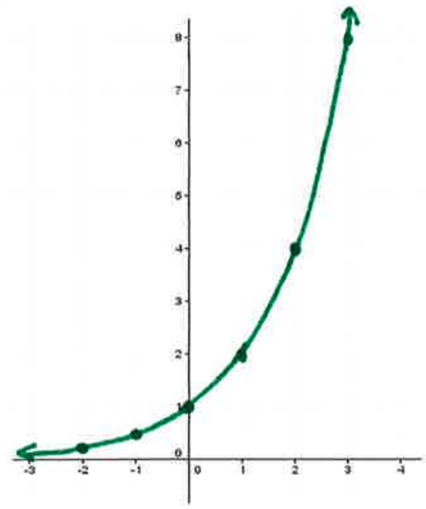
Number of amoebas = (initial population) • (growth factor)$^{\text{Number of splits}}$

Graphs of Exponential Functions $\qquad$ **$h(t) = 2^t$**

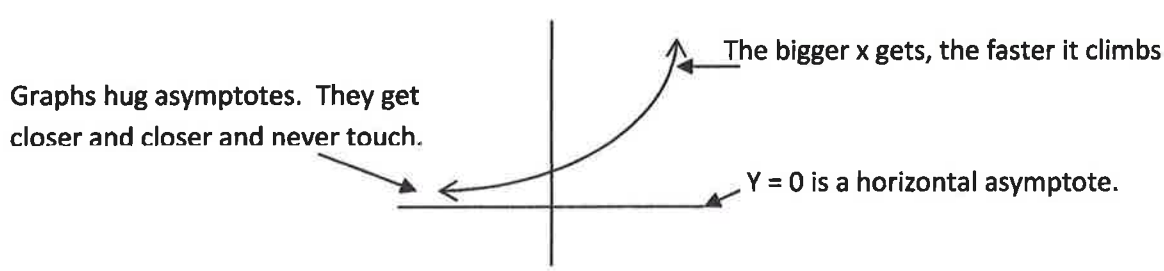
Let's graph it! To make it easier, let's graph $y = 2^x$, since we're use to graphing (x,y).

Well, when we have no idea what something looks like, we have to plot points:

X	$Y = 2^x$
-2	$2^{-2} = ¼$
-1	$2^{-1} = 1/2$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$



This is the basic shape of the graph of an exponential function: $\qquad y = a^x$ where $a > 1$
This function is an illustration of "exponential growth".



Adapted from a lesson created by Lauren Brooks, Enloe High School, WCPSS 2012 Teacher of the Year.

More Alien Monster and Amoeba Encounters

For each alien encounter below, write the explicit equation in function notation and then solve.

1. An alien amoeba colony is growing exponentially and had a population of 10,000 when it was first observed. Three hours later, the population was 80,000. What was the population six hours after it was first observed? What will be the population in 10 hours? In 24 hours?

$\frac{80000}{10000} = 8$

multiplies by 8 every 3 hours

6 hours $\frac{6}{3} = 2$

$y = 10000(8)^2$

$y = 640000$

10 hours $\frac{10}{3} = 3.\overline{3}$

$y = 10000(8)^{3.\overline{3}}$

$y = 10240000$

24 hours $\frac{24}{3} = 8$

$y = 10000(8)^8$

$y = 1.68 \times 10^{11}$

In each case, what did you have to do to the number of hours to find your exponent?

Divide by 3.

2. The population of the alien city, found on the dark side of the moon, has grown at a rate of 3.2% each year for the last 10 years. If the population 10 years ago was 25,000, what is the population today?

$y = 25000(1.032)^{10}$

$y = 34256$

3. In 2010, the population of a monster city, called Halloween Town, was 50 monsters. Since then the population has increased at a constant rate of 25% each year. Assuming this rate of increase stays constant, what will the monster population of Halloween Town be in 4 years? In 20 years?

$y = 50(1.25)^x$

4 years

$y = 50(1.25)^4$

$y = 122$

20 years

$y = 50(1.25)^{20}$

$y = 4337$

4. A population of alien bacteria grows by 35% every hour. If the population begins with 100 alien specimens, how many are there after 6 hours? How many will there be in 18 hours?

$y = 100(1.35)^x$

6 hours

$y = 100(1.35)^6$

$y = 605$

18 hours

$y = 100(1.35)^{18}$

$y = 22182$

5. The population in the town of Alien Acres is presently 42,500. The town has been growing at a steady rate of 2.7%. Find the number of years ago that the population was 30,000.

$y_1 = 42500(1.027)^x$

$y_2 = 30000$

-13.07 years ago from point of intersection