Exponential Decay: Depreciation Problems

Most cars lose value each year by a process known as **depreciation.** You may have heard before that a new car loses a large part of its value in the first 2 or 3 years and continues to lose its value, but more gradually, over time. That is because the car does not lose the same **amount** of value each year, it loses approximately the same **percentage** of its value each year. What kind of model would be useful for calculating the value of a car over time? exponential

Let us look at an example of depreciation: Suppose the value of car when new is $20,000 and it depreciates at a rate of 20% each year. What is the percentage rate of depreciation each year?

20% gone, so 80% remains

The percentage rate of depreciation is 20%, which means that 80% of the value of the car remains every year. We can calculate this percentage rate by subtracting 20% from 100% in order to calculate the value remaining of 80% each year.

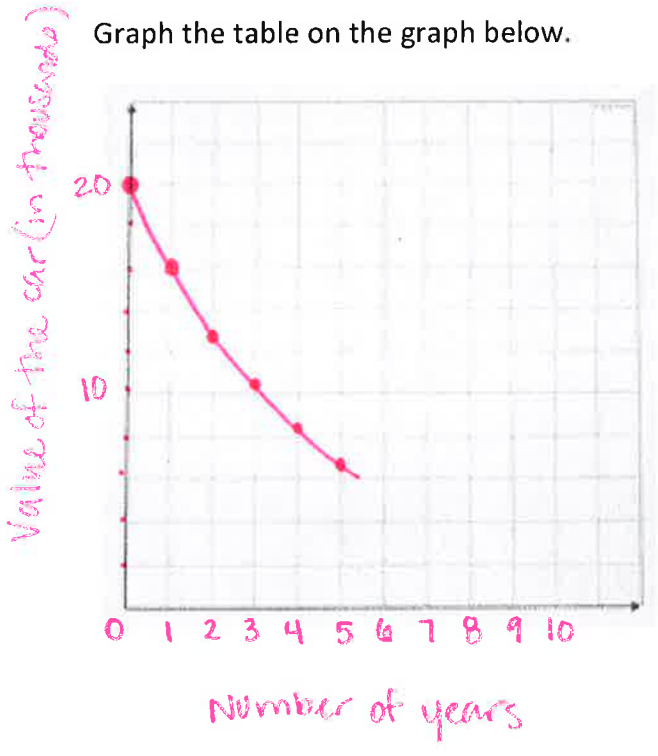
What is the initial value of the car? $ 20 000

What is the percentage rate? 100% - 20% = 80 % each year = .8

Let us look at the depreciation data over a 5 year period of time (rounded to the nearest dollar).

Number of Years	0	1	2	3	4	5
Value of the Car	20,000	16,000	12800	10240	8,192	6554

Graph the table on the graph below.



Write an explicit equation for the data in order to calculate the value of deprecation for any year.

Y = initial value (1 – percentage rate of depreciation)^{time}

Y = 20,000(1 - 0.20)^{x}

Y = 20,000(0.80)^{x}

Use this equation to find the depreciated value of the car for year 8.

Y = 20,000(0.80)^{8} = $ 3355.44

When will the depreciated value of the car be worth $5000? 6.21 years

Estimate this value to the nearest tenth of a year with your calculator by inputting the equation in your calculator ~~and looking at the table.~~ in Y_1 & 5000 in Y_2 Then find the point of intersection

YOUR TURN

Matt bought a new car at a cost of $25,000. The car depreciates approximately 15% of its value each year.

a.) What is the percentage **rate of depreciation** for the value of this car?
(Remember that the percentage rate of depreciation is $0 < b < 1$.)

100 - 15 = 85%

b.) Write an equation to model the decay value of this car.

y = $25000(.85)^{x}$

where *y* is the value of the car; *x* is the number of years since new purchase

c.) What will the car be worth in 10 years?

$4921.86

Guided Practice with Depreciation Problems

Other vehicles and equipment depreciate over time as well, such as trucks, boats, tractors, computer equipment, etc. Let us look at other items which depreciate over time.

The cost of a new truck is $32,000. It depreciates at a rate of 15% per year. This means that it loses 15% of each value each year.

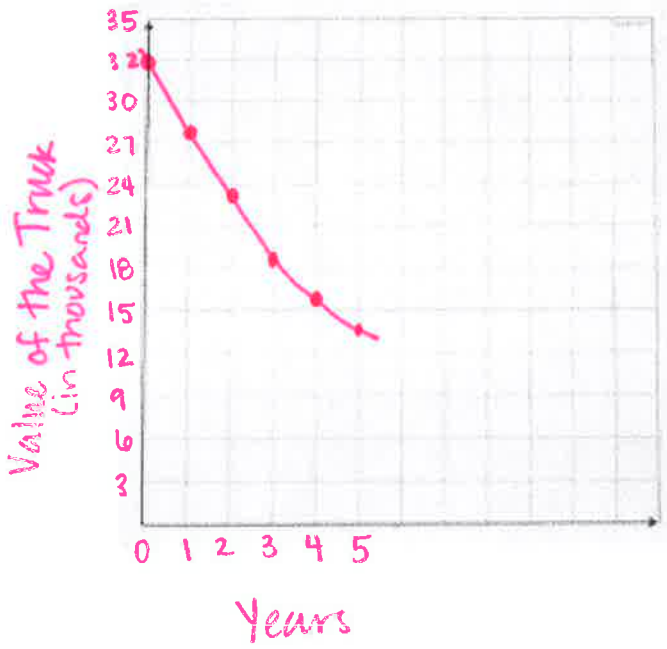
Tasks:

- Draw the graph of the truck's value against time in year.
- Find the formula that gives the value of the truck in terms of time.
- Find the value of the truck when it is four years old.
- Estimate when the truck will be worth half of its value (about $16,000).

Let's start by making a table of values. To fill in the values we start with 32,000 at time $t = 0$. Then we multiply the value of the car by 85% for each passing year. (Since the car looses 15% of its value, which means that it keeps 85% of its value). Remember that 85% means that we multiply by the decimal 0.85.

Number of Years	0	1	2	3	4	5
Value of the Truck	32,000	27200	23120	19652	16704	14199

Graph the data on the coordinate grid below. Remember to label your axes.



Now let us write the equation for the data.

Initial value: 32 000

Percentage rate of depreciation: 85%

Equation: $y = 32000(.85)^x$

Use the equation to determine the value of the truck when it is 4 years old.

Value of the 4 year old truck: 16704

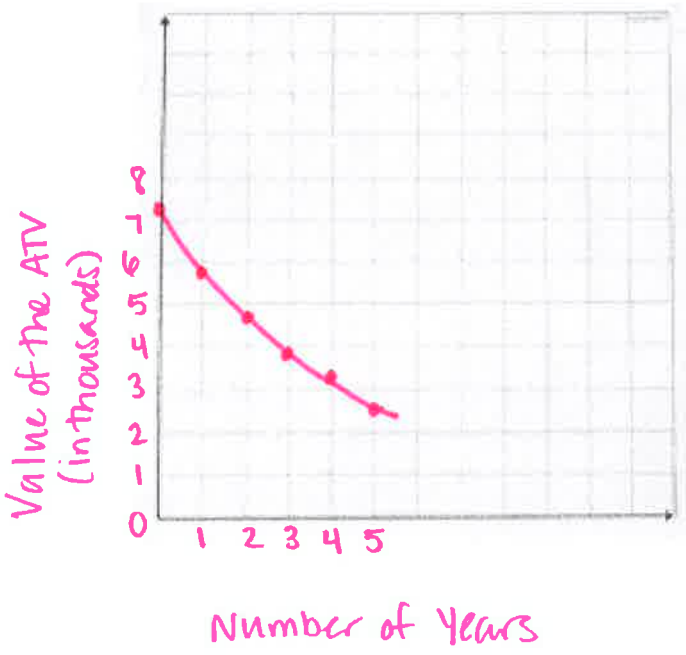
Compare this value with the value in the data table. It should be the same value if your equation is correct.

Use the table, graph, equation, or graphing calculator to estimate the time it will take for the truck to worth half of its initial value. 4.27

Try a few more:

1) The cost of a new ATV (all-terrain vehicle) is $7200. It depreciates at 18% per year. Draw the graph of the vehicle's value against time in years. Find the formula that gives the value of the ATV in terms of time. Find the value of the ATV when it is ten year old.

Number of Years	0	1	2	3	4	5
Value of the ATV	7200	5904	4841	3970	3255	2669



$y = 7200(.82)^x$

2) A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

a. Write an exponential decay equation for the value of equipment.

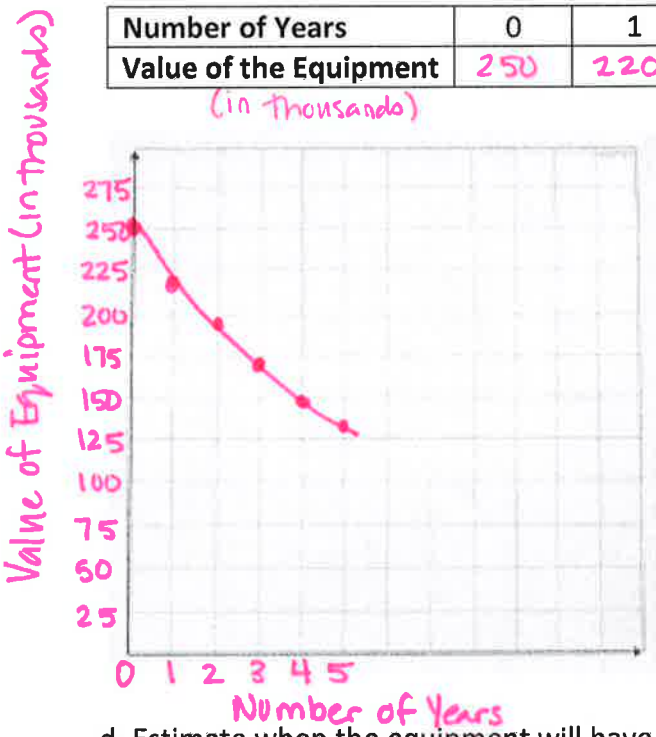
$y = 250000(.88)^x$

b. What is the value of equipment after 5 years?

131,933

c. Make a table and graph the model.

Number of Years	0	1	2	3	4	5
Value of the Equipment	250	220	193.6	170.4	149.9	131.9



d. Estimate when the equipment will have a value of $70,000.

9.96 years

Independent Practice with Depreciation Problems

Use equations, graph, or tables to find the solutions to the problems below.

1) A computer valued at $6500 depreciates at the rate of 14.3% per year. Write a function that models the value of the computer. Find the value of the computer after three years.

$y = 6500(.857)^x$

$y = 6500(.857)^3 = \$4091.25$

2) A new truck that sells for $29,000 depreciates 12% each year. Write a function that models the value of the truck. Find the value of the truck after 7 years.

$y = 29000(.88)^x$

$y = 29000(.88)^7 = \$11,851.59$

3) A new car that sells for $18,000 depreciates 25% each year. Write a function that models the value of the car. Find the value of the car after 4 years.

$y = 18000(.75)^x$

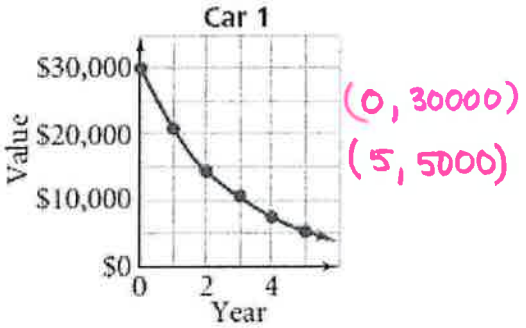
$y = 18000(.75)^4 = \$5695.31$

4) You purchased a car for $19,500. The car will depreciate at a rate of 12% each year. Write a formula to represent the value of the car after x number of years. Find the value of the car after 4 years.

$y = 19500(.88)^x$

$y = 19500(.88)^4 = \$11694.06$

5) Each graph below shows the expected decrease in a car's value over the next five years. Write a function to model each car's depreciation. Determine which car will be worth more after 10 years.

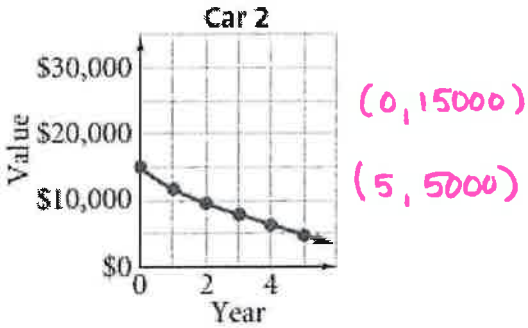


(0, 30000)

(5, 5000)

$y = 30000(.69)^x$

In 10 years $733.86



(0, 15000)

(5, 5000)

$y = 15000(.80)^x$

in 10 years $1610.61