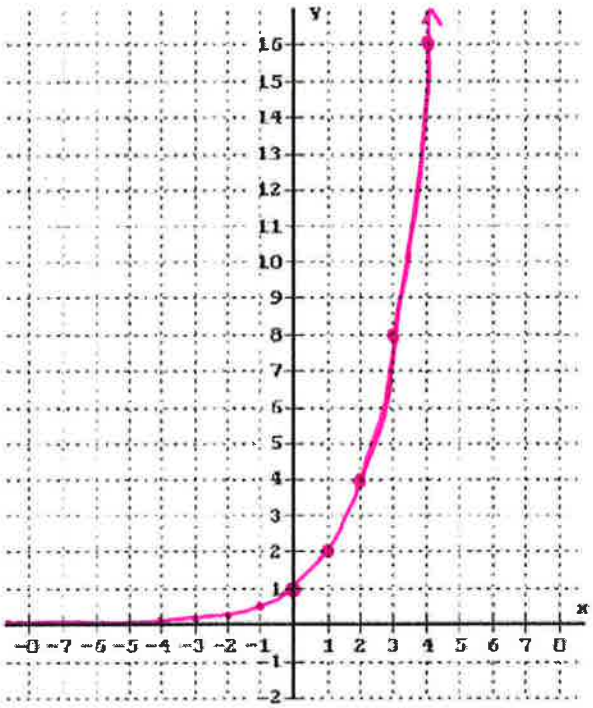
Translation Investigation

Graph the Exponential Function: $f(x) = 2^x$

1. Complete the Table.

x	$y = 2^x$
5	$2^5 = 32$
4	16
3	8
2	4
1	2
0	1
−1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
−2	1/4
−3	1/8
−4	1/16
−5	1/32
10	1024
−10	1/1024

2. Graph the function.



3. Where does the graph of $y = 2^x$ cross the y-axis? That is, find the y-intercept(s)?

(0, 1)

4. Where does the graph of $y = 2^x$ cross the x-axis? That is, find the x-intercept(s)?

It doesn't cross the x-axis.

Did you notice that this exponential function gets *really* really really close to crossing the x-axis, but it never actually does? That's because the x-axis ($y = 0$) is an asymptote for the function $y = 2^x$. Asymptotes are lines that a function gets infinitely close to, but never crosses.

5. State the domain.

all real numbers

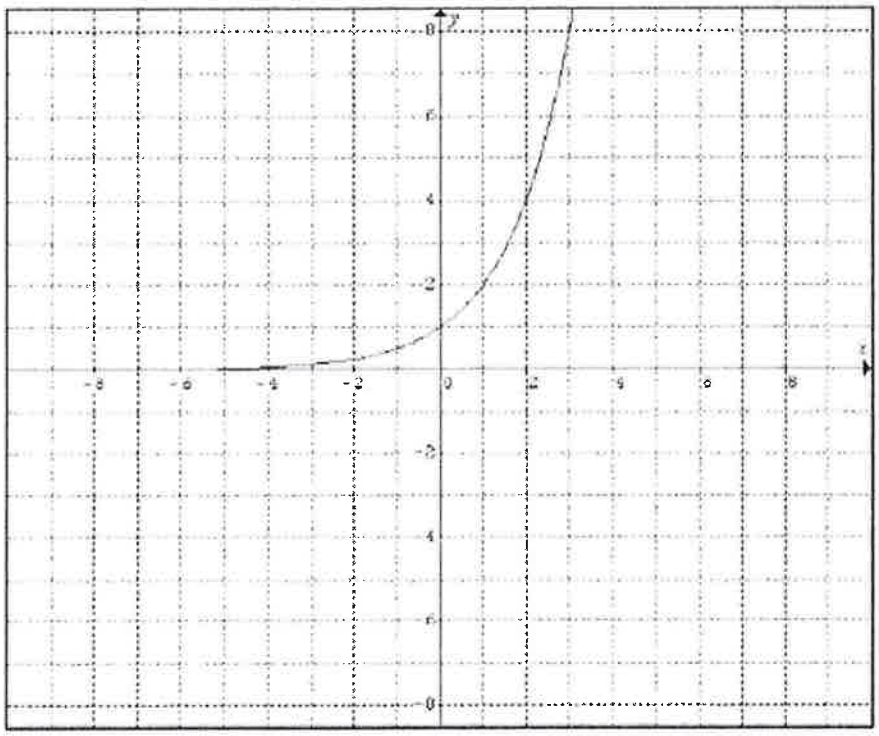
6. State the range.

$y > 0$

7. Describe what the ends of the exponential graph do – we call this end behavior. Do all exponential graphs have the same end behavior?

The graph goes to the left along the x-axis forever and on the right it goes up forever. Some exponentials are flat on the left and go down instead of up.

Given the graph of the exponential function $f(x) = 2^x$:



Graph the following functions on the coordinate plane above. After graphing each function, describe the change from the parent function $y = 2^x$.

8. Graph: $y = 2^x - 5$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	-4.969	-4.938	-4.875	-4.75	-4.5	-4	-3	-1	3	11	27

What change occurred from the parent graph $y = 2^x$?

It moved down 5

What is the new asymptote?

$y=-5$

Where is the y-intercept now?

$(0,-4)$

What is the new domain?

all real numbers

What is the new range?

$y>-5$

9. Graph $y = 2^{x-5}$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	.00098	.00195	.00391	.00781	.01563	.03125	.0625	.125	.25	.5	1

What change occurred from the parent graph $y = 2^x$?

It moved right 5

What is the new asymptote?

$y=0$

Where is the y-intercept now?

$(0, .03125)$

What is the new domain?

all real numbers

What is the new range?

$y>0$

10. Graph $y = 2^{x-5} - 4$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	-3.999	-3.998	-3.996	-3.992	-3.984	-3.969	-3.938	-3.875	-3.75	-3.5	-3

What change occurred from the parent graph $y = 2^x$?

it moved down 4 and right 5

What is the new asymptote?

$y = -4$

Where is the y-intercept now?

$(0, -3.969)$

What is the new domain?

all real numbers

What is the new range?

$y > -4$

11. Graph: $y = 2^x + 3$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	3.0313	3.0625	3.125	3.25	3.5	4	5	7	11	19	35

What change occurred from the parent graph $y = 2^x$?

it moved up 3

What is the new asymptote?

$y = 3$

Where is the y-intercept now?

$(0, 4)$

What is the new domain?

all real numbers

What is the new range?

$y > 3$

12. Graph $y = 2^{x+3}$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	.25	.5	1	2	4	8	16	32	64	128	256

What change occurred from the parent graph $y = 2^x$?

it moved left 3

What is the new asymptote?

$y = 0$

Where is the y-intercept now?

$(0, 8)$

What is the new domain?

all real numbers

What is the new range?

$y > 0$

13. Graph $y = 2^{x+3} + 4$ by making a table.

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values	4.25	4.5	5	6	8	12	20	36	68	132	260

What change occurred from the parent graph $y = 2^x$?

it moved up 4 and left 3

What is the new asymptote?

$y = 4$

Where is the y-intercept now?

$(0, 12)$

What is the new domain?

all real numbers

What is the new range?

$y > 4$

14. Describe in your own words how to make a horizontal shift with an exponential function.

add or subtract to the x in the exponent to move left (+) or right(-

What does the general exponential function look like (Use h to represent your horizontal change)?

$y = ab^{x+h}$

How does a horizontal shift affect the asymptote? Do you think this is always the case?

A horizontal shift does not affect the asymptote ever.

How does a horizontal shift affect the y-intercept?

The y-intercept increases when it shifts left and decreases when it shifts right

How does a horizontal shift affect the domain?

A horizontal shift does not affect the domain.

How does a horizontal shift affect the range?

A horizontal shift does not affect the range.

15. Describe in your own words how to make a vertical shift with an exponential function.

add or subtract to the end of the function to move up (+) or down (-)

What does the general exponential function look like (use k to represent your vertical change)?

$y = ab^x + k$

How does a vertical shift affect the asymptote? Do you think this is always the case?

A vertical shift moves the asymptote up or down "k" units.

How does a vertical shift affect the y-intercept?

A vertical shift moves the y-intercept up or down "k" units.

How does a vertical shift affect the domain?

A vertical shift does not affect the domain.

How does a vertical shift affect the range?

A vertical shift moves the range up or down "k" units.

16. Describe in your own words how to make both a vertical and horizontal shift with an exponential function.

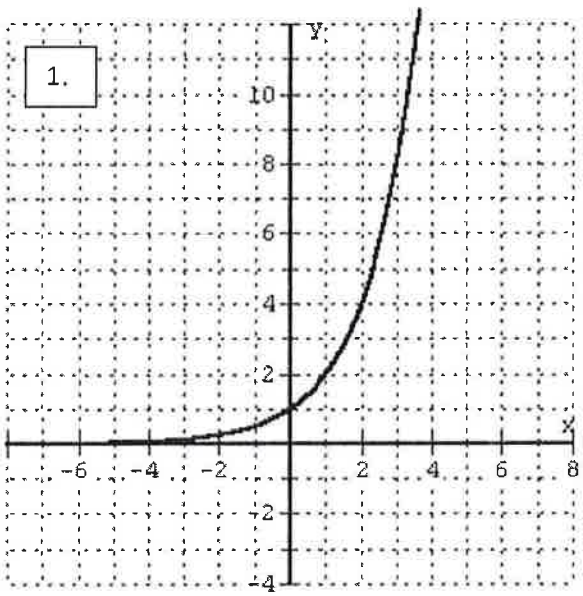
To make a horizontal & vertical change, add or subtract in the exponent and at the end of the equation.

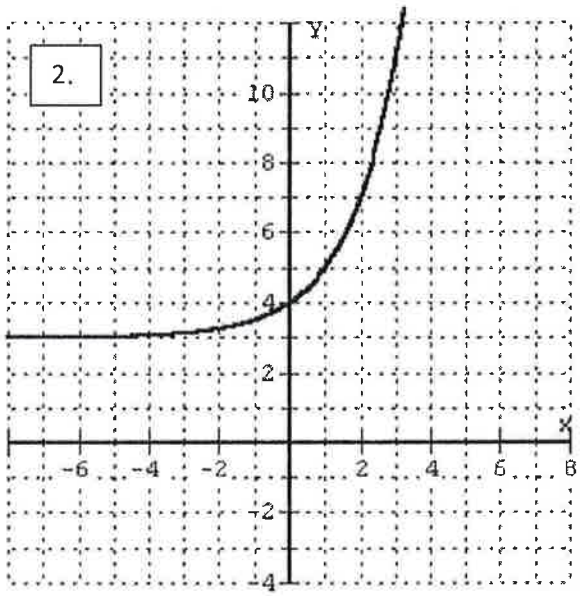
What does the general exponential function look like (Use h for your horizontal change and k for your vertical change)?

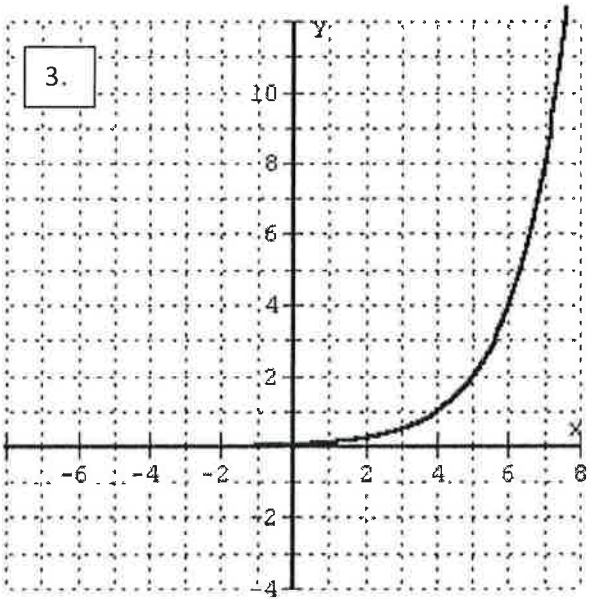
$y = ab^{x+h} + k$

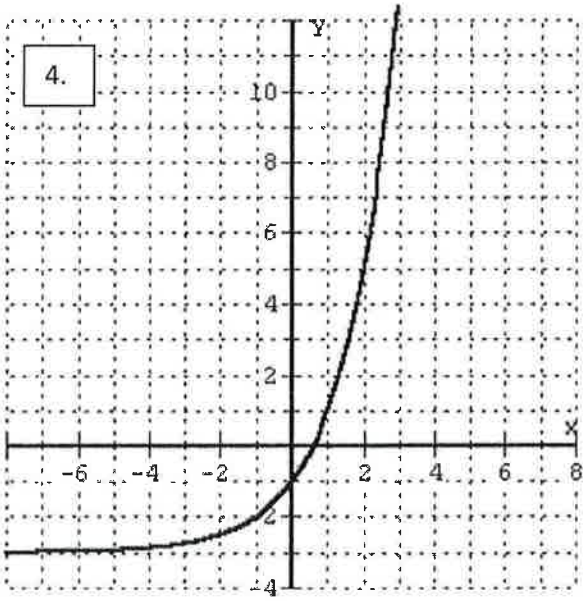
Independent Practice with Translations of Exponential Functions

Given the parent graph below and on the left, write the equation of this function and the remaining 3 functions by looking at the translations from the parent graph.









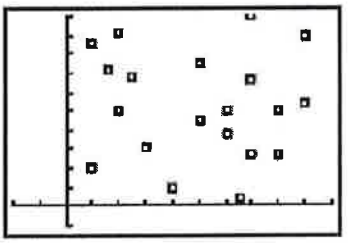
Write your equations here:

1. $y = 2^x$ 2. $y = 2^x + 3$ 3. $y = 2^{x-4}$ 4. $y = 2^{x+1} - 3$

Hit To Win

Your task is to try to "hit" as many points as possible on a coordinate grid by changing the values of *A* and *B* in either linear or exponential equations. To "hit" a point the line or exponential must hit in the center of the square that locates the point. You are allowed to use a total of 4 functions and they may be either linear or exponential. The person in your group to "hit" the most points is the winner.

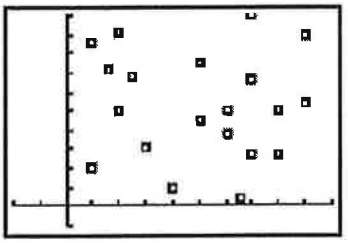
Circle your equation and record your values in the box below. Sketch your graphs on the coordinate grid. Try this one as target practice.



Equation	Your values	Number of Points Hit
$Y = Ax + B$ or $Y = AB^x$	A = B =	
$Y = Ax + B$ or $Y = AB^x$	A = B =	
$Y = Ax + B$ or $Y = AB^x$	A = B =	
$Y = Ax + B$ or $Y = AB^x$	A = B =	

Round One

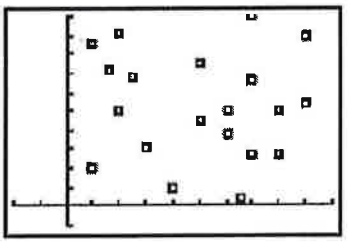
Now that you tried the practice round, let's ramp it up a bit and use the exponential equation $Y = AB^x + C$. Circle your equation and record your values in the box below. Sketch your graphs on the coordinate grid.



Equation	Your values	Number of Points Hit
$Y = Ax + B$ or $Y = AB^x + C$	A = B = C =	
$Y = Ax + B$ or $Y = AB^x + C$	A = B = C =	
$Y = Ax + B$ or $Y = AB^x + C$	A = B = C =	
$Y = Ax + B$ or $Y = AB^x + C$	A = B = C =	

Round Two

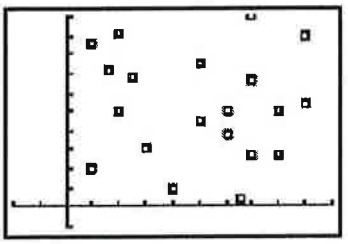
Let's try a change in strategy by using the exponential equation $Y = AB^{x+c}$. Circle your equation and record your values in the box below. Sketch your graphs on the coordinate grid.



Equation	Your values	Number of Points Hit
$Y = Ax + B$ or $Y = AB^{x+c}$	A = B = C =	
$Y = Ax + B$ or $Y = AB^{x+c}$	A = B = C =	
$Y = Ax + B$ or $Y = AB^{x+c}$	A = B = C =	
$Y = Ax + B$ or $Y = AB^{x+c}$	A = B = C =	

Round Three

Let's try a change in strategy by using the exponential equation $Y = AB^{(x+c)} + D$. Circle your equation and record your values in the box below. Sketch your graphs on the coordinate grid.



Equation	Your values	Total Points
$Y = Ax + B$ or $Y = AB^{(x+c)} + D$	A = B = C = D =	
$Y = Ax + B$ or $Y = AB^{(x+c)} + D$	A = B = C = D =	
$Y = Ax + B$ or $Y = AB^{(x+c)} + D$	A = B = C = D =	
$Y = Ax + B$ or $Y = AB^{(x+c)} + D$	A = B = C = D =	

Suggested classroom set-up:

I would put the students in groups of 4. Allow the students to partner up, so that each group of 4 has 2 teams. Allow the students to solve the problems in teams of two and then have them compare their points for each round.

Debrief the activity by asking for the largest number of points for each round and have the "winning teams share out their equations.

Each round forces the students to create different equations in order to gain more points. There are many solutions for the rounds. Focus on the shifting of the linear and exponential equations and how the equations are changed from round to round.

Feel free to allow graphing calculators. The points plotted in each round include: (1,2) (1, 8.5) (2.5) (2.9) (1.5, 7.5) (5,7) (3,3) (4,1) (5, 4.5) (5, 7.5) (6, 3.75) (6,5) (6.5, 0) (7, 2.5) (7, 6.5) (7.10) (8.2.5) (8, 5) (9, 5.5) (9,9)

Adapted from "Wrapping It All Up" Student Worksheet, Texas Instruments Incorporated, 2004, pp. 106-107.