Population Growth Problems

*MORE SPACE PROVIDED ON STUDENT VERSION

One practical application of exponential growth is predicting growth of various populations. You have already looked at problems involving population growth when looking at the Alien and Monster lesson, and other problems encountered in lessons or homework. Some of the growth factors were whole numbers and some were fractional. In this lesson we will explore more fractional growth factors.

Aussie Rabbits

In 1859, a small number of rabbits were introduced to Australia by English Settlers. The rabbits had no natural predators in Australia, so they reproduced rapidly and quickly became a serious problem for sheep and cattle, eating the grasses intended for them.

In the mid-1900s, there were more than 300 million rabbits in Australia. The damage they caused cost Australian agriculture $600 million per year. There have been many attempts to curb Australia's rabbit population. In 1995, a deadly rabbit disease was deliberately spread, reducing the rabbit population by about half. However, because rabbits are developing immunity to the disease, the effects of this measure may not last.

If biologists had counted the rabbits Australia in the years after they were introduced, they might have collected data like these:

Growth of Rabbit Population

Time (Year)	Population
0	100
1	180
2	325
3	583
4	1,050

A. The table shows the rabbit population growing exponentially.

1. What is the growth factor? Explain how you found your answer. 1.8 divide $\frac{180}{100}$
2. Assume this growth pattern continued. Write an equation for the rabbit population *p* for any year *n* after the rabbits are first counted. Explain what the numbers in your equation represent. $p = 100(1.8)^n$ 100 = start 1.08 = growth factor
3. How many rabbits will there be after 10 years? How many will there be after 25 years? After 50 years? 10 years: 35705 25 years: 240886512 50 years: 5.8×10^{14}
4. In how many years will the rabbit population exceed one million?
   between 15 & 16 years

B. Suppose that, during a different time period, the rabbit population could be predicted by the equation $p = 15(1.2)^n$, where p is the population in millions, and n is the number of years.

1. What is the growth factor?

   1.2

2. What was the initial population?

   15

3. In how many years will the population double from the initial population?

   almost 4 years (3.80 years)

4. What will the population be after 3 years? After how many more years will the population at 3 years double?

   There will be 52 rabbits in 6.82 years.

   3 years ~26 rabbits

5. What will the population be after 10 years? After how many more years will the population at 10 years double?

   There will be 186 rabbits in 13.81 years.

   10 years ~93 rabbits

6. How do the doubling time for parts (3) – (5) compare? Do you think the doubling time will be the same for this relationship no matter where you start to count?

   Based on this pattern, it seems likely that it will take about 3.8 years to double!

The yearly growth factor for the table above is about 1.8. Suppose the population data fit the equation $p = 100(1.8)^n$ exactly. Then its table would look like the one below.

Rabbit Population Growth

Time (Year)	Population
0	100
1	100 • 1.8 = 180
2	180 • 1.8 = 324
3	324 • 1.8 = 583.2
4	583.2 • 1.8 = 1,049.76

The growth factor of 1.8 is the number by which the population for year n is multiplied to get the population for the next year, $n + 1$. You can think of the growth factor in terms of a percent change.

Change the growth factor into a percentage and you get 180 %

Here's a formula for percent change (make sure you follow the order of operations!)

$$\frac{\text{NEXT} - \text{NOW}}{\text{NOW}} = \frac{180-100}{100} = \frac{324-180}{180} = \frac{583.2-324}{324} = \frac{1049.76-583.2}{583.2} = 0.80 = 80\%$$

Okay, but why is the growth factor 1.8 (180%) if the percent change is only 80%?

Because 100% of the previous year's rabbits are still there, plus 80% more.

How can the equation $p = 100(1.8)^n$ be written as a NOW-NEXT equation?

NEXT = NOW(1.8), start at 100

YOUR TURN

Let's look at another population . . . the wolves of northern Michigan.

In parts of the United States, wolves are being reintroduced to wilderness areas where they had become extinct. Suppose 20 wolves are released in northern Michigan, and the yearly growth factor for this population is expected to be 1.2.

1. Make a table showing the projected number of wolves at the end of each of the first six years.

x	y
0	20
1	24
2	29
3	35
4	41
5	50
6	60

Rounded to the nearest whole wolf.

2. Write a NOW-NEXT equation that models the growth of the wolf population.

NEXT = NOW(1.2) start at 20

3. Write an explicit equation in function form that models the growth of the wolf population.

$y = 20(1.2)^x$

4. Using either equation, how long will it take for the new wolf population to exceed 100?

8.83 years

5. Using either equation, how many wolves will there be in 10 years?

124

15 years?

308

25 years?

1908

Population Growth and Other Word Problems

The Elk Population

1) The table show that the elk population in a state forest is growing exponentially. What is the growth factor? Explain. The growth factor is about 1.9 because any term divided by the previous is ~1.9

Growth of Elk Population

Time (Year)	Population
0	30
1	57
2	108
3	206
4	391
5	743

$\frac{57}{30} = 1.9$

$\frac{108}{57} = 1.89$

$\frac{206}{108} = 1.91$

2) Suppose this growth pattern continues. How many elk will these be after 10 years? How many elk will there be after 15 years?

10 years → 18397　　15 years → 455538

3) Write a NOW-NEXT equation you could use to predict the elk population *p* for any year *n* after the elk were first counted.

NEXT = NOW (1.9)　start at 30

4) Use this equation to write an explicit equation in function notation to predict the elk population *p* for any year *n* after the elk were first counted.

$p = 30(1.9)^n$

5) In how many years will the elk population exceed one million?

17 years

For problems 6 and 7, write a NOW-NEXT equation and an explicit equation in function notation before find the solution(s) to the problems.

6) Suppose there are 100 trout in a lake and the yearly growth factor for the population is 1.5. How long will it take for the number of trout to double?

NEXT = NOW (1.5)　start at 100

$y = 100(1.5)^x$

There will be 200 trout in 1.71 years

7) Suppose there are 500,000 squirrels in a forest and the growth factor for the population is 1.6 per year. Write an equation you could use to find the squirrel population *p* in *n* years.

$p = 500000(1.6)^n$

8) Currently, 1,000 students attend East Garner IB Magnet Middle School. The school can accommodate 1,300 students. The school board estimates that the student population will grow by 5% per year for the next several years. $y = 1000(1.05)^x$

a) In how many years will the population outgrow the present building?

5.38 years

b) Suppose the school limits its growth to 50 students per year. How many years will it take for the population to outgrow the school?

linear function
$y = 50x + 1000$

6 years

9) Suppose that, for several years, the number of radios sold in the U.S. increased by 3% each year.

a) Suppose one million radios sold in the first year of this time period. About how many radios sold in each of the next 6 years? $y = 1000000(1.03)^x$

1000000, 1030000, 1060900, 1092727, 1125509, 1159274

b) Suppose only 100,000 radios sold in the first year. About how many radios sold in each of the next 6 years?

100000, 103000, 106090, 109273, 112551, 115927

10) Suppose a movie ticket costs about \$7, and inflation causes ticket prices to increase by 4.5% a year for the next several years. $y = 7(1.045)^x$

a) At this rate, how much will tickets cost 5 years from now?

\$8.72

b) How much will a ticket cost 10 years from now?

\$10.87

c) How much will a ticket cost 30 years from now?

\$26.22

d) When will a ticket cost \$25?

28.9 years

Independent Practice with Population Growth and Other Word Problems

Write a NOW-NEXT equation and explicit equation in function notation to assist in solving the following problems.

1) Omar made the following calculation to predict the value of his baseball card collection several years from now:

Value = \$130 • 1.07 • 1.07 • 1.07 • 1.07 • 1.07

a) What initial value, growth rate, growth factor, and number of years is Omar assuming?

$a_1 = 130$ $r = 7\% = .07$ $b = 1.07$

b) Write the equation modeling this problem in function notation.

$V(x) = 130(1.07)^x$

c) If the value continues to increase at this rate, how much would the collection be worth after 8 years?

$V(8) = 130(1.07)^8 = \$223.34$

2) Carlos, Latanya, and Mila work in a biology laboratory. Each of them is responsible for a population of mice. The growth factor for Carlos's population of mice is 8/7. The growth factor for Latanya's population of mice is 3. The growth factor for Mila's population of mice is 125%.

a) How mice are reproducing fastest?

Latanya's

b) Whose mice are reproducing slowest?

Mila's

Carlos	1.14	14%
Latanya	3	200%
Mila	1.25	25%

3) A worker currently receives a yearly salary of \$20,000.

a) Find the dollar values of a 3%, 4%, and 6% raise for this worker.

3%	4%	6%
\$600	\$800	\$1200

b) Find the worker's new annual salary for each raise in part a.

3%	4%	6%
\$20600	\$20800	\$21200

c) You can find the new salary after a 3% raise in two ways:

\$20,000 + 3% of \$20,000 OR 103% of \$20,000

Explain why these two methods give the same result.

$20000 + .03(20000)$ $1.03(20000)$

$(1+.03)(20000)$

$1.03\ (20000) \longrightarrow \$20,600$

4) Kwan cuts lawns every summer to make money. One of her customers offers to give her a 3% raise next summer and a 4% raise the summer after that. Kwan says she would prefer to get a 4% raise next summer and a 3% raise the summer after that. She claims she will earn more money this way. Is she correct? Explain why or why not.

$x(1.03)(1.04)$ OR $x(1.04)(1.03)$ no, she'll get

$1.0712x$ OR $1.0712x$ 107.12% more either way

5) In 1990, the population of the U.S. was about 250 million and was growing exponentially at a rate of about 1% per year.

a) At this growth rate, what will the population of the U.S. be in the year 2010? 20 years

$y = 250(1.01)^{20} = 305$ million

b) At this rate, how long will it take the population to double?

69.7 years

c) Do you think the predictions in parts a and b are accurate? Explain.

Probably not, people live longer now, so the population will begin increasing more quickly.

d) The population in 2000 was about 282 million. How accurate was the growth rate?

The model predicts 276 million, so it's only 6 million off.

6) The Greens bought a condominium for $83,000. Assuming that its value will appreciate 6% per year, how much will the condo be worth in 5 years when the Greens are ready to move? $y = 83000(1.06)^5 = \$111072.72$

7) Ten years ago, Mr. and Mrs. Boyce bought a house for $96,000. Their home is now worth $125,000. Assuming a constant growth rate, what was the annual rate of appreciation?

$y_1 = 96000(x)^{10}$ $x = 1.027$ 2.7% growth

$y_2 = 125000$

8) In 2000, the number of people worldwide living with HIV/AIDS was estimated at more than 36 million. That number was growing at an annual rate of about 15%.

a) Make a table showing the projected number of people around the world living with HIV/AIDS in each of the ten years after 2000, assuming the growth rate remains 15% per year.

2000	36 million	2005	72.4 million
2001	41.4 million	2006	83.3 million
2002	47.6 million	2007	95.8 million
2003	54.8 million	2008	110.1 million
2004	63.0 million	2009	126.6 million
		2010	145.6 million

b) Write two different kinds of rules that could be used to estimate the number of people living with HIV/AIDS at any time in the future.

NEXT = NOW(1.15) $y = 36(1.15)^x$

start at 36

(Problem 8 Continued)

c) Use the rules from part b to estimate the number of people living with HIV/AIDS in 2015.

292.9 million

d) What factors might make the estimate of part c an inaccurate forecast?

A cure could be discovered, healthcare & education could improve, or there could be an epidemic.

9) Hypothermia is a life-threatening condition in which body temperatures fall well below the norm of 98.6°F. However, because chilling causes normal body functions to slow down, doctors are exploring ways to use hypothermia as a technique for extending time of delicate operations like brain surgery.

The following table gives experimental data illustrating the relationship between body temperature and brain activity.

Body Temperature (in °F)	50	59	68	77	86	98.6
Brain Activity (% Normal)	11	16	24	37	52	100

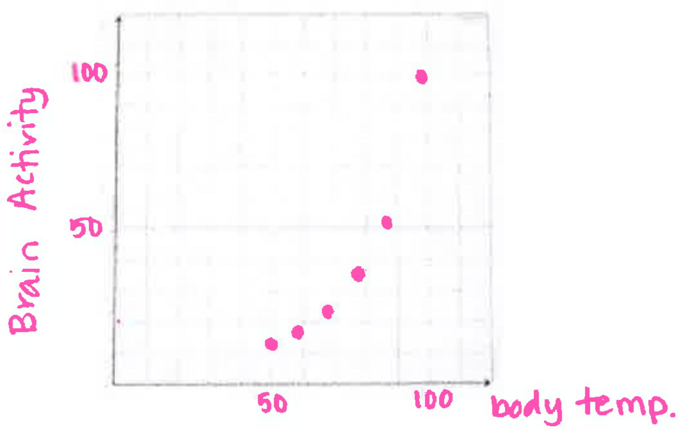
a) Plot the table data and find an explicit equation that models the pattern in these data relating brain activity level to body temperature. Then express the same relationship with an equivalent NOW-NEXT rule.

exponential regression

$y = 1.13(1.05)^x$

NEXT = NOW(1.05)

start at 1.13



b) Use your rules to estimate the level of brain activity at a body temperature of 39°F, the lowest temperature used in surgery experiments on pigs, dog, and baboons.

39°F → 6.55% brain activity

using regression table

c) Find the range of body temperatures at which brain activity is predicted to be about 75% of normal levels.

~93°F → 75% brain activity

y_1 = regression equation

y_2 = 75%

point of intersection