Bacteria Growth

If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose a single bacterium lands on your tooth and starts multiplying by a factor of 4 every hour.

1. Complete the table below to model the bacteria growth over several hours.

Hours	Expanded Form	Exponential Form (fill in the blank)	Evaluated Form
0	1	$1 \cdot 4^{0}$	1
1	$1 \cdot 4$	$1 \cdot 4^{1}$	4
2	$1 \cdot 4 \cdot 4$	$1 \cdot 4^{2}$	16
3	$1 \cdot 4 \cdot 4 \cdot 4$	$1 \cdot 4^{3}$	64
4	$1 \cdot 4 \cdot 4 \cdot 4 \cdot 4$	$1 \cdot 4^{4}$	256
5	$1 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$	$1 \cdot 4^{5}$	1024

1. Graph the data in the table below. Be sure to label your graph and axes.
2. Is this graph linear or exponential?

   exponential

3. What is the initial value in this situation?

   1

3. What is the common ratio in this situation?

   4

4. What do the first & third columns have in common?

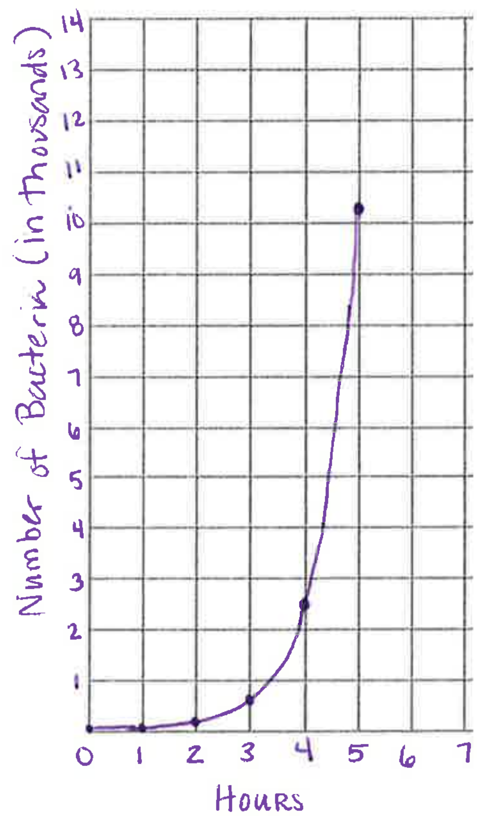
   The number of hours matches the exponent

5. What are the similarities and differences between the entries in the third column of the table?

   All are multiplied by 1 and have a base of 4, but the exponents on the 4 increase based on the hours that pass.

6. Use the pattern you saw in the third column to predict how many bacteria there will be after 10 hours. Continue the pattern in the table or on the graph to verify your answer.

   1048576



7. If y is the number of bacteria after x hours, write a rule that will allow you to calculate how many bacteria there are at any time.

   $y = 1 \cdot 4^{x}$

8. Write the *NOW-NEXT* form to show the pattern of growth.

   NEXT = 4 • *NOW*

y = the number of bacteria produced in that hour
x = the number of hours
r = the common ratio or rate of change
a_1 = the initial term of the sequence or the starting point

Use the above information to write the explicit form of the exponential function $y = a_1 \bullet r^x$. Notice how similar it is to the NOW-NEXT recursive form.

$$\begin{array}{ccccc} NEXT & = & NOW & \bullet & r \\ \downarrow & & \downarrow & & \downarrow \\ y & = & a_1 & \bullet & r^x \\ \downarrow & & \downarrow & & \downarrow \\ y & = & 1 & \bullet & 4^x \end{array}$$

The *NEXT* and y components both represent the number of bacteria generated during the hour. The *NOW* and a_1 both represent the starting point and r is the rate of change or the common ratio, which is 4 in this example.

9. After how many hours will there be at least 1,000,000 bacteria in the colony?

   9.96 $\approx$ 10 hours

   $y_1 = 1 \cdot 4^x$
   $y_2 = 1000000$

   point of intersection shows 1000000 at 9.96 h

10. Suppose that instead of 1 bacterium, 50 bacteria land in your mouth. Write an explicit equation which describes the number of bacteria y in this colony after x hours.

    $y = 50 \cdot 4^x$

11. What is different in this equation from the equation in step 7?

    The number multiplied times the power (the "multiplier") is 50.

12. Using your new equation, determine the number of bacteria in the colony after 8 hours and after 10 hours.

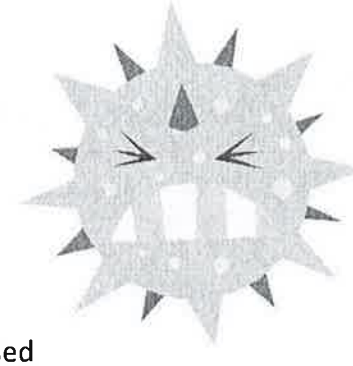
    8 hours → 3276800

    10 hours → 52428800

13. Which method for determining the number of bacteria is easier for you? Using a table, graph, NOW-NEXT, or explicit equation? Explain.

    Answers will vary, but most likely explicit because you don't have to repeat steps repeatedly.

Guided Practice: More Bacteria

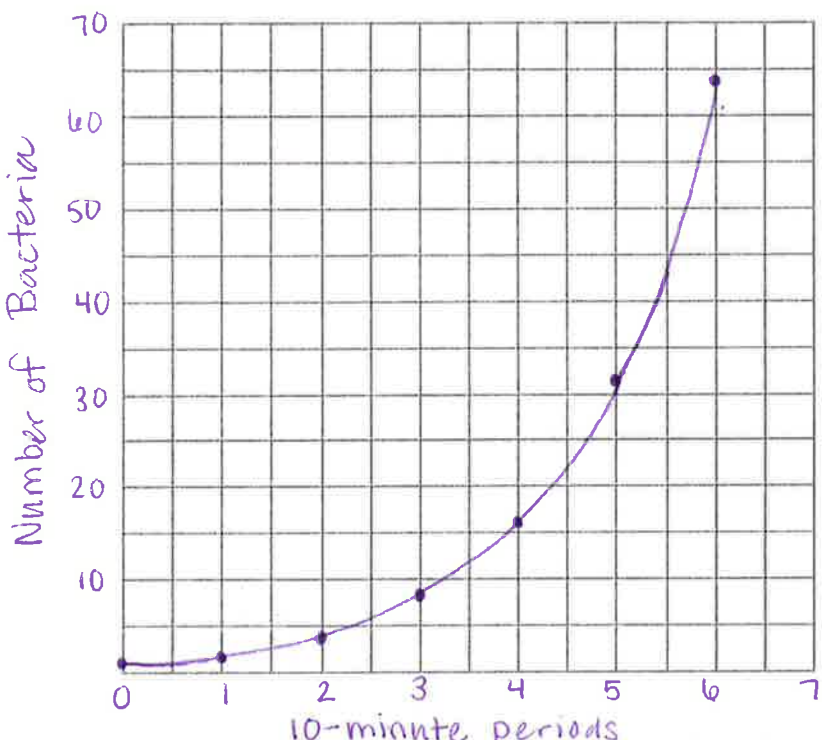


The bacteria E. coli often causes illness among people who eat the infected food. Suppose a single E. coli bacterium in a batch of ground beef begins doubling every 10 minutes.

1. Complete the table below to determine how many bacteria there will be after 10, 20, 30, 40, and 50 minutes have elapsed (assuming no bacteria die).

10-min Period	0	1	2	3	4	5
Number of Bacteria	1	2	4	8	16	32

2. Graph the data on the table. Be sure to title your graph and label your axes.



3. Write two rules that can be used to calculate the number of bacteria in the food after any number of 10-minute periods.

$$NEXT = NOW \cdot \underline{2}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$y = a_1 \cdot r^x$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$y = \underline{1} \cdot \underline{2}^x$$

4. What is the initial value? 1

5. What is the common ratio? 2

6. How many times would the bacteria double in 2 hours?

2 hours = 120 minutes = 12 10-minute intervals

7. Use your rule(s) to determine the number of bacteria after 2 hours.

$y = 2^{12} = 4096$ bacteria

8. When will the number of bacteria reach at least 100,000?

between 160 & 170 minutes (166.1 minutes)

Students at a high school conducted an experiment to examine the growth of mold. They set out a shallow pan containing a mixture of chicken broth, gelatin, and water. Each day, the students recorded the area of the mold in square millimeters. The students wrote the exponential equation $m = 50(3^d)$ to model the growth of the mold. In this equation, m is the area of the mold in square millimeters after d days.

9. What is the area of the mold at the start of the experiment?

50 mm^2

10. What is the growth factor or common ratio?

3

11. What is the area of the mold after 5 days?

$50 \cdot 3^5 = 12150 \text{ mm}^2$

12. On which day will the area of the mold reach 6,400 mm^2?

during the 4th day (4.41 days)

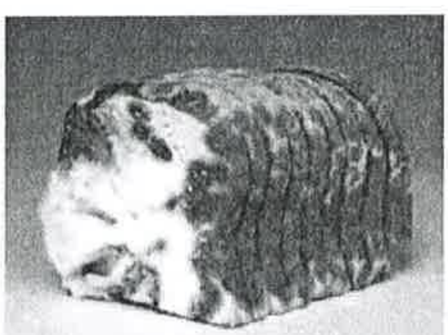
13. An exponential equation can be written in the form $y = a(b^x)$, where a and b are constant values.

   a. What value does b have in the mold equation? What does this value represent?

   3 is the common ratio

   b. What value does a have in the mold equation? What does this value represent?

   50 is the initial value



Lesson adapted from *Growing, Growing, Growing Exponential Relationships*, Connected Mathematics 2, Pearson, 2009.

Independent Practice: Killer Plants

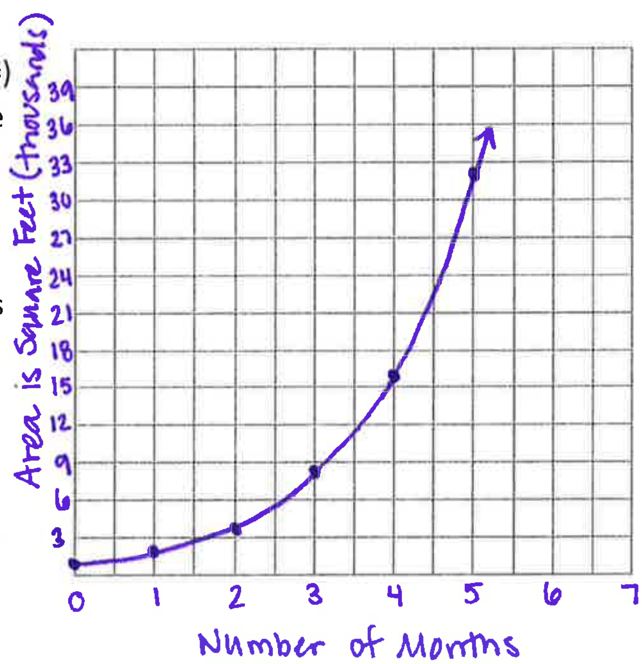
Ghost Lake is a popular site for fishermen, campers, and boaters. In recent years, a certain water plant has been growing on the late at an alarming rate. The surface area of Ghost Lake is 25,000,000 square feet. At present, 1,000 square feet are covered by the plant. The Department of Natural Resources estimates that the area is doubling every month.

1. Complete the table below.

Number of Months	0	1	2	3	4	5
Area Covered in Square Feet	1,000	2000	4000	8000	16000	32000

2. Use the data to graph the situation. Be sure to label your axes and title your graph.

✓



3. Write 2 equations (NOW-NEXT and y =) to represent the growth pattern of the plant on Ghost Lake.

NEXT= NOW·2, Start at 1000

$y = 1000 \cdot 2^x$

4. Explain what information the variables and numbers in your equations represent.

2 represents the doubling

1000 is the initial value

5. How much of the lake's surface will be covered with the water plant by the end of a year?

January = month 0

December = month 12

$y = 1000 \cdot 2^{12} = 4096000$ square feet

6. How much of the lake's surface was covered by the water plant 6 months ago?

$y = 1000 \cdot 2^{-6} = 15.625$ square feet

7. In how many months will the plant completely cover the surface of the lake?

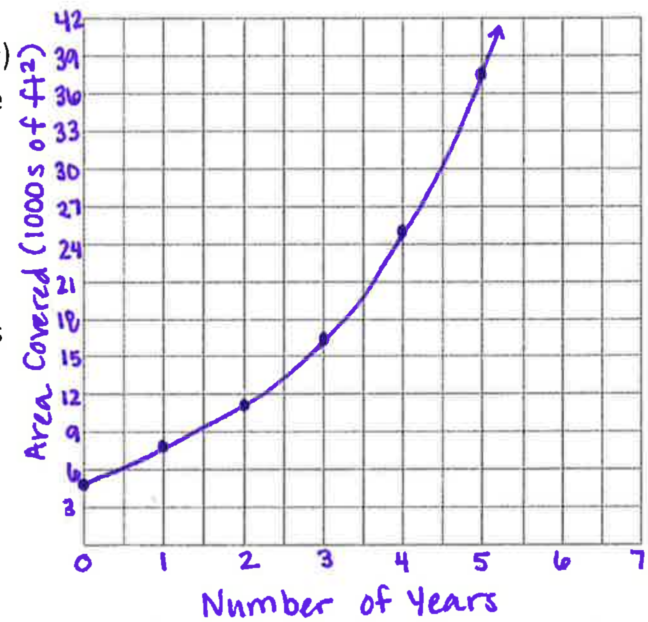
between 14 & 15 months (14.6 months)

Loon Lake has a "killer plant" problem similar to Ghost Lake. Currently, 5,000 square feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.

8. Complete the table to show the area covered by the plant for the next 5 years.

Number of Years	0	1	2	3	4	5
Area Covered in Square Feet	5,000	7500	11250	16875	25313	37969

9. Graph the data. Be sure to label your axes and title your graph.



10. Write 2 equations (NOW-NEXT and y =) to represent the growth pattern of the plant on Ghost Lake.

NEXT = NOW(1.5) Start at 5000

$y = 5000(1.5)^x$

11. Explain what information the variables and numbers in your equations represent.

1.5 is the growth factor

5000 is the initial value

12. How much of the lake's surface will be covered with the plant by the end of 7 years?

$y = 5000(1.5)^7$ = 85429.7 square feet

13. The surface area of the lake is approximately 5 acres. How long will it take before the lake is completely covered if one acre is 43,560 square feet?

between 9 & 10 years (9.10 years)

FYI: One acre is about the size of a football field with the end zones cut off

Adapted from Growing, Growing, Growing Exponential Relationships, Connected Mathematics 2, Pearson, 2009.