Word Problem Review Worksheet

Use your knowledge of exponential functions to answer the following questions. Show your work, especially equations written to assist in finding solutions.

1. Which is the best investment if the money in each case is invested for three years?
   a. \$5,000 at 8% compounded monthly \$6351.19
   b. \$5,000 at 8.2% compounded annually \$6333.62
   c. \$5,000 at 8.1% compounded semiannually \$6344.87

2. The population of a bacteria culture doubles after 1.5 hours. An experiment begin siwth 620 bacteria. Determine the number of bacteria after
   a. 3 hours 2480
   b. 6 hours 4920
   c. 10 hours 62488
   d. 1 day 40632320
   e. 3 days 1.75×10^{17}
   f. 1 week 2.32×10^{53}

3. The half-life of a radioactive material is about 2 years. How much of a 5-kg sample of this material would remain after
   a. 4 years 1.25 kg
   b. 3 years 1.77 kg
   c. 5.5 years 0.743 kg
   d. 18 months 2.97 kg

4. The population of Littleton is currently 23,000. Assume that Littleton's exponential growth rate is 2% per year.
   a. Copy and complete the table by predicting the population for the next six years.

Time (years)	0	1	2	3	4	5	6
Population	23,000	23460	23929	24408	24896	25394	25902

   b. Graph the data.
   c. Create the equation to model the equation. $y = 23000(1.02)^x$
   d. Use your equation to predict the population in 10 years. 28037
   e. Use your graph to estimate how long it will take the population will reach 30,000. 13.41 years
   f. Predict the population of Littleton after 10 years if the growth rate is 3%. 30910

5. A population, P, is increasing exponentially. At time $t = 0$, the population is 35,000. In 10 years, the population is 44,400.
   a. Find a in $P = k(a)^t$. $a = 1.024$ (Exp Reg)
   b. Use the value of a that you calculated, write an equation that models the population, P, after t years. $P = 35000(1.024)^t$
   c. Using your equation, find when the population reaches 100,000. 44.13 years

6. A bacteria culture starts with 3,000 bacteria and grows to a population of 12,000 after 3 hours. $y = 3000(1.587)^x$ (Exp Reg)
   a. Find the doubling period. 1.5 hours
   b. Find the population after *t* hours. $y = 3000(1.587)^t$
   c. Determine the number of bacteria after 8 hours. 120952
   d. Determine the number of bacteria after 1 hour. 4762

7. The half-life of caffeine in a child's system when a child eats or drinks something with caffeine in it is 2.5 hours. How much caffeine would remain in a child's body if the child ate a chocolate bar with 20 mg of caffeine 8 hours before?
   $y = 20(0.5)^{8/2.5} = 2.18$ mg

8. Twelve grams of tritium decays to 9.25 grams in 2.5 years. Use a method to estimate the half-life of tritium. 6.658 years

9. A radioactive form of uranium has a half-life of 2.5×10^5 years.
   a. Find the remaining mass of 1 gram sample after t years. $y = 1(0.5)^{(t/2.5 \times 10^5)}$
   b. Determine the remaining mass of this sample after 5000 years. 0.986g

10. The half-life of carbon-14 is about 5370 years. What percent of the original carbon-14 would you expect to find in a sample after 2500 years?
    $(0.5)^{2500/5370} = 72.4\%$

11. An old stamp is currently worth \$60. The stamp's value will grow exponentially 15% per year. $y = 60(1.15)^x$
    a. What will the value of the stamp be in 8 years? \$183.54
    b. When will the value of the stamp be worth 3 times the initial value? 7.86 years

12. A photocopier, which originally costs \$500,000, depreciates exponentially by 10% each year. $y = 500000(0.90)^x$
    a. What will the photocopier's value be worth in 5 years? \$295245
    b. When will the photocopier's value be \$175,000? 9.96 years

13. After an accident at a nuclear power plant, which caused a radiation leak, the radiation level at the accident was 950 roentgens. Five hours later, the radiation level was 800 roentgens. Radiation levels decay exponentially. Find the rate of decay.
    $b = .96$ $r = 100|1 - .96| = 4\%$

14. Annie bought a new car for \$35,000 and sold it 5 years later for \$18, 475. Assume that the value of the vehicle depreciates exponentially. Calculate the rate of depreciation per year. $b = .88$ $r = 100|1 - .88| = 12\%$

15. Mark invests \$500 in a savings plan that pays interest, which is compounded monthly. At the end of 10 years, his initial investment is worth \$909.70. What interest rate did the plan pay?

$b = 1.06$ $\quad r = 100|1 - 1.06| = 6\%$

16. An exponential function is expressed in the form $y = a(b)^x$. How can you tell whether the relation represents growth or decay?

growth $b > 1$
decay $0 < b < 1$

17. The population of a small town increases exponentially. In 1999, the population was 16,000 and in 2002 it was 60,000. What will the population be in 2010?

2040000

18. In 1996, Ontario's population was about 10.7 million. Ontario's population will be about 13.7 million in 2016. $y = 10.7(1.012)^x$
    a. Calculate the annual growth rate of Ontario's population. $r = 100|1 - 1.012| = 1.2\%$
    b. What would Ontario's population have been in 1980? 8.78 million
    c. What have you assumed for part a and part b?

We assume the growth is exponential and the growth factor stays the same.

19. During an archaeological dig, Selma found a tool that resembled a small hatchet with a wooden handle.
    a. Carbon-14 has a half-life of about 5370 years. Explain what this means.
    b. Create an equation that relates the percent of carbon-14 remaining to the tool's age. Explain what each part of the equation represents.
    c. Explain how you can tell from the equation that the amount of carbon-14 is decreasing.
    d. Explain how you can tell from the equation that the amount of carbon-14 is decreasing exponentially.

a) Half of the Carbon-14 would be gone in 5370 years.

b) $y = a(0.5)^{t/5370}$

y = remaining amount of Carbon-14
a = initial amount of Carbon-14
0.5 = growth factor for half-life scenarios
t = time in years
5370 = half-life of Carbon-14

c) The growth factor is between 0 and 1, so this is an exponential decay problem.

d) We know that it is decreasing exponentially because the independent variable is an exponent. Also, the carbon-14 is decreasing by 50% each year, so 0.5 is a common ratio, not a common difference.

Exponential Functions Unit Review

1. Records at the Universal Video store show that sales of new DVDS are greatest in the first month after the release date. In the second month, sales are usually only about one-third of sales in the first month. Sales in the third month are usually only about one-third of sales in the second month, and so on.

   a. If Universal Video sells 31886460 copies of one particular DVD in the first month after its release, how many copies are likely to be sold in the second month? In the third month? Use the table below to help you answer the questions.

Number of Months	0	1	2	3	4	5
Number of DVD Sales	31886460	10628820	3542940	1180980	393660	131220

   b. What NOW-NEXT and "y = " rules predict the sales in the following months?

   NEXT = NOW(1/3) start at 31886460 $y = 31886460\left(\frac{1}{3}\right)^x$

   c. Use your equations to predict how many DVDs are in the 12th month?

   60 DVDs

   d. In what month are sales likely to first be fewer than 5 copies?

   15th month

   e. How would your answers to parts a – d change for a different DVD that has first month sales of 26572050 copies?

Number of Months	0	1	2	3	4	5
Number of DVD Sales	26572050	8857350	2952450	984150	328050	109350

   NOW-NEXT rule: NEXT = NOW(1/3)

   Y = $26572050(1/3)^x$

   Number of DVDs in the 12th month: 50 DVDs

   First month in which fewer than 5 copies are sold: 15th month

2. Find the next three terms in each sequence. Identify each as arithmetic, geometric, or neither. For each arithmetic or geometric sequence, find the common difference or common ratio. Then write a NOW-NEXT rule to describe the sequence.
   a. 14, 11, 8, 5, 2 . . . arithmetic, d = -3, NEXT = NOW - 3
   b. 3,000, 300, 30, 3 . . . geometric, r = 0.1 NEXT = NOW(0.1)
   c. 5, 6, 8, 11, 15, 20 . . . neither

3. Tell whether each situation produces an arithmetic sequence, a geometric sequence, or neither.
   a. The temperature rises at the rate of 0.75°F per hour. arithmetic
   b. A person loses 2 lbs each month. arithmetic
   c. A toadstool doubles in size each week. geometric
   d. A person receives a 6% raise each year. geometric

4. Graph each function on graph paper and state the y-intercept.

   a. $Y = 2^x - 3$ b. $y = 3^{x+1}$ c. $y = (1/3)^x$

5. You may have heard of athletes being disqualified from competitions because they have used anabolic steroid drugs to increase their weight and strength. These drugs are dangerous and leave the body slowly. With an injection of the steroid cyprionate, about 90% of the drug and its by-products will remain after a second day, and so on. Suppose that an athlete tries steroids and injects a dose of 100 mg of cyprionate.

   a. Make a table showing the amount of the drug remaining at various times.

Number of Days	0	1	2	3	4	5
Amount of Cyprionate	100	90	81	72.9	65.61	59.04

   b. Make a plot of the data in part a on your graph paper and write a short description of the pattern shown.

   c. Write two rules that describe the amount of steroid in the blood.

   NOW-NEXT rule: NEXT = NOW (0.9)   start at 100

   Y = $100(0.9)^x$

   d. Use one of the rules in part c to estimate the amount of steroid left after 0.5 days and 8.5 days.

   0.5 days → 94.8 mg   8.5 days → 40.8 mg

   e. Estimate, to the nearest tenth of a day, the half-life of cyprionate.

   6.6 days

   f. How long will it take the steroid to be reduced to only 1% (1 mg) of its original level in the body?

   44 days

6. For each of the following rules, decide whether the function represented is an example of: an increasing linear function, a decreasing linear function, an exponential growth function, an exponential decay function, or neither a linear or exponential function.

   a. $Y = 5(0.4^x)$ exp. decay   b. NEXT = 5 • NOW exp. growth

   c. $Y = 5 - 0.4x$ dec. linear   d. NEXT = NOW – 5 dec. linear

   e. $Y = 5/x$ neither   f. NEXT = 0.4 • NOW exp. decay

7. In 2000, the number of people worldwide living with HIV/AIDS was estimated at more than 36 million. That number was growing at an annual rate of about 15%.

   a. Make a table showing the projected number of people around the world living with HIV/AIDS in each of the ten years after 2000, assuming the growth rate remains 15% per year.

Years after 2000	0	1	2	3	4	5	6	7	8	9	10
AIDS Cases (in millions)	36	41.4	47.6	54.8	63.0	72.4	83.3	95.8	110.1	126.6	145.6

   b. Write two different kinds of rules that could be used to estimate the number of people living with HIV/AIDS at any time in the future.

   NEXT = now (1.15) start at 36

   Y = $36(1.15)^x$

   c. Use the rules from part b to estimate the number of people living with HIV/AIDS in 2015.

   292.93 million

   d. What factors might make the estimate of part c an inaccurate forecast?

   advances in medicine/prevention/education

8. Write each of the following expressions in a simpler equivalent exponential form.

   a. $7^4 \bullet 7^9 =$ 7^{13}

   b. $x \bullet x^4 =$ x^5

   c. $(x^2)^3 =$ x^6

   d. $(t^3)^4 =$ t^{12}

   e. $(5x^3y^4)(4x^2y) =$ $20x^5y^5$

   f. $(c^5d^3)^2 =$ $c^{10}d^6$

   g. $(\frac{2}{5})^3 =$ $\frac{2^3}{5^3} = \frac{8}{125}$

   h. $\left(\frac{1}{3}\right)^{-3} =$ $3^3 = 27$

   i. $\frac{5^7}{5^5} =$ $5^2 = 25$

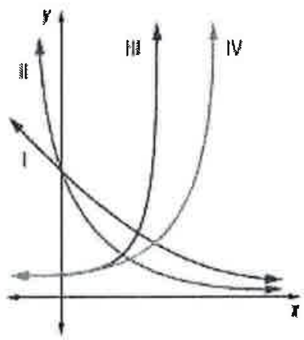
   j. $\frac{t^5}{t^2} =$ t^3

   k. $\frac{30x^3y^2}{6xy} =$ $5x^2y$

   l. $\frac{ab^2}{a^4b} =$ $\frac{b}{a^3}$

9. The graphs, tables, and rules below model four exponential growth and decay situations. For each graph, there is a matching table and a matching rule. Use what you know about the patterns of exponential relations to match each graph with its corresponding table and rule. In each case, explain the clues that can be used to match the items without any use of a graphing calculator or computer.

Graphs



Tables

A

x	1	2	3	4
y	40	16	6.4	2.56

II, 2

B

x	0	1	2	3	4
y	10	30	90	270	810

III, 4

C

x	1	2	3	4
y	60	36	21.6	12.96

I, 1

D

x	0	1	2	3	4
y	10	20	40	80	160

IV, 3

Rules

(1) $y = 100(0.6^x)$

(2) $y = 100(0.4^x)$

(3) $y = 10(2^x)$

(4) $y = 10(3^x)$

10. Exponential functions can be expressed by a rule relating x and y values and by a rule relating NOW and NEXT values.
    a. Write a general rule for an exponential function y = ab^x
    b. Write a general rule relating NOW and NEXT for an exponential function.
    next = now · b, start at a
    c. What do the parts of the rules tell you about the problem situation?
    a is your initial value, b is the multiplier
    d. How do you decide whether a given exponential function rule will describe growth or decay, and why does your decision rule make sense?
    If $b > 1$, it's growth. If $0 < b < 1$, it's decay. This makes sense because if you multiply by a number greater than 1, you get a larger number. If you multiply by a number between 0 & 1, you get a smaller number.

Adapted from *Core-Plus Mathematics*, Glencoe McGraw-Hill: Columbus, OH, 2008.