Linear Functions versus Exponential Functions

The aim of this investigation is to develop students' ability in recognizing data patterns likely to be modeled well by exponential growth functions. A further goal is to utilize graphing calculator experimentation to find a good regression model. Students should think analytically about the data being modeled as well as to use estimation and calculator-based tools.

Wolf Populations in the Midwest

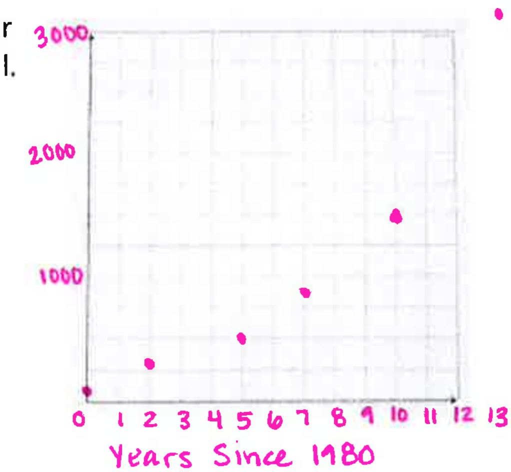
Suppose that census counts of Midwest wolves began in 1980 and produced these estimates for several different years:

Time Since 1980 (in years)	0	2	5	7	10	13
Estimated Wolf Population	100	300	500	900	1,500	3,100

1) Plot the wolf population data on paper and decide whether a linear or exponential function seems likely to match the pattern of growth well.

exponential

$y = 138.4(1.28)^x$



2) Use your graphing calculator to find both linear and exponential regression models for the given data pattern. Make sure you have turned your Diagnostics On (2[nd] 0).

Linear
y = $212.3x - 242.8$
r^2 = 0.865

Exponential
y = $138.4(1.28)^x$
r^2 = 0.970

3) What do the numbers in the linear and exponential function rules from part 2 suggest about the pattern of change in the wolf population? $y = a + bx$

Linear
b = adding 212.3 each year
a = population in 1980 → -243

Exponential
a = population in 1980 was 138
b = population is multiplied by 1.28 each year

4) Which model do you think best fits the data? Why?

exponential → it grows slowly at first and then more quickly

Also r^2 is closer to 1.

5) Use the model for wolf population growth that you believe to be best to calculate population estimates for the missing years (1981, 1983, 1984, 1986, 1988, 1989, 1991, and 1992).

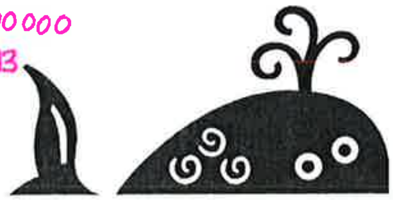
Time Since 1980 (in years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Estimated Wolf Population	100 138	177	300 226	289	370	500 473	605	900 774	990	1265	1,500 1618	2069	2646	3,100 3384

6) Use your model to give population estimates for the year 2000, 2005, and 2010. When will the population reach an estimated 500,000 wolves?

2000 → 19290
2005 → 66280
2010 → 227738

after 33.3 years there will be 500000 wolves - so in 2013

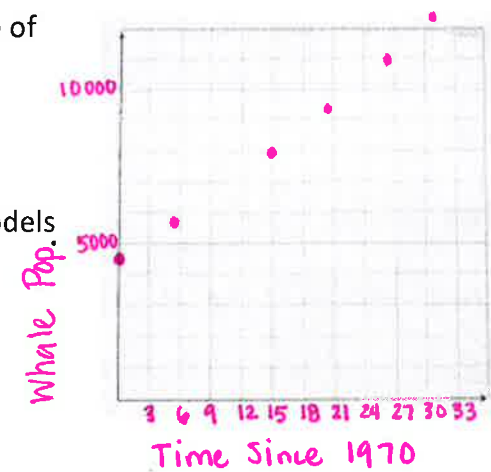
Alaskan Bowhead Whales



Suppose that census counts of Alaskan Bowhead Whales began in 1970 and produced these estimates for several different years:

Time Since 1970 (in years)	0	5	15	20	26	31
Estimated Whale Population	4700	5,800	8,000	9,300	11,000	12,300

7. Plot the given whale population data on paper and decide which type of function seems likely to match the pattern of growth well.



8. Use your calculator to find both linear and exponential regression models for the data pattern.

Linear $y = a + bx$
y = $245x + 4554$
r^2 = 0.997

Exponential $y = ab^x$
y = $4884(1.03)^x$
r^2 = 0.993

9. Which model do you think best fits the data? Why?

This data appears to be linear - the growth seems constant, and r^2 is a little closer to 1.

10. What do the numbers in the linear and exponential function rules from problem 8 suggest about patterns of change in the whale population?

Linear
b = adding 245 whales a year
a = there are 4554 whales in 1970

Exponential
a = there are 4884 whales in 1970
b = population is multiplied by 1.03 each year

11. Use the model for whale population growth that you believe to be the best to calculate population estimates for the years 2002, 2005, and 2010.

2002 → 12397

2005 → 13133

2010 → 14358

12. When will the whale population reach 25,000?

y_1 → linear regression
y_2 → 25000

point of intersection shows 25000 whales in 83.4 years, so 2053

Summarize the Mathematics

In the problems of this investigation, you studied ways of finding function models for growth patterns that could only be approximated by one of the familiar types of functions.

13. How do you decide whether a data pattern is modeled best by a linear or exponential function?

graphing the data to see whether it's a line or curve and if it's hard to tell, looking at the r^2 value to find which is closest to 1.

14. What do the numbers *a* and *b* in a linear function $y = a + bx$ tell about patterns in the $\boxed{\text{graph}}$ of the function?

b tells you the slope

a tells you the y-intercept

15. What do the numbers *a* and *b* in a linear function $y = a + bx$ tell about patterns in a $\boxed{\text{table}}$ of (x, y) values for the function?

b tells you the common difference - what's added or subtracted

a tells you the starting value (where $x=0$)

16. What do the numbers *a* and *b* in a exponential function $y = a(b^x)$ tell about patterns in the $\boxed{\text{graph}}$ of the function?

a tells you the y-intercept

b tells you the factor by which the previous y-value is multiplied

17. What do the numbers *a* and *b* in a exponential function $y = a(b^x)$ tell about patterns in a $\boxed{\text{table}}$ of (x, y) values for the function?

a tells you the starting value (where $x=0$)

b tells you how much to multiply y by to get the next y-value

18. What strategies are available for finding a linear or exponential function that models a linear or exponential data pattern?

a value find the y-intercept (where $x=0$)

b value for linear $\frac{y_2-y_1}{x_2-x_1}$

for exponential $\sqrt[(x_2-x_1)]{\frac{y_2}{y_1}}$

When using real world data, you seldom get the exact values in any model, but the calculator's linear & exponential regressions can provide the best model based on the data

Independent Practice with Linear Functions versus Exponential Functions

Exponential functions, like linear functions, can be expressed by rules relating x and y values and by rules relating NOW and NEXT y values when an x value increases in steps of 1. Compare the patterns of (x, y) values produced by these functions: $y = 2(3^x)$ and $y = 2 + 3x$ by completing these tasks.

1. For each function write another rule using NOW and NEXT that could be used to produce the same pattern of (x, y) values.

   $y = 2(3)^x$ NEXT = NOW · 3
   Start at 2

   $y = 2 + 3x$ NEXT = NOW + 3
   Start at 2

2. How would you describe the similarities and differences in the relationships of x and y in terms of their function graphs, tables, and rules?
   a. Similarities and differences of function graphs

      both have a y-intercept of (0,2)
      both are increasing

      one is going up by a constant of 3 and the other by a multiple of 3

   b. Similarities and differences of function tables

      both contain the point (0,2)
      if x increases by 1, y is increased by 3 in linear & multiplied by 3 in exponential

   c. Similarities and differences of function rules

      both begin with y = 2...
      one is multiplied by 3^x. In the other 3x is added

3. When will the exponential function "overtake the linear function"? Will this happen all of the time or just some of the time? Explain your thoughts.

   The exponential overtakes the linear at .54.
   The exponential will always overtake the linear because of the multiplication.

U.S. Presidential Elections

The following table shows the number of votes cast in a sample of U.S. Presidential elections between 1840 and 2004

Year of Election	Major Party Candidates	Total Votes Cast
1840	Harrison vs. Van Buren	2,411,118
1860	Lincoln vs. Douglas	4,685,030
1880	Garfield vs. Hancock	9,218,951
1900	McKinley vs. Bryan	14,001,733
1920	Harding vs. Cox	26,757,946
1940	Roosevelt vs. Wilkie	49,752,978
1960	Kennedy vs. Nixon	68,836,385
1980	Reagan vs. Carter	86,515,221
2000	Bush vs. Gore	105,405,100
2004	Bush vs. Kerry	122,267,553

4. Find rules for what you think are the best possible linear and exponential models of the trend relating votes cast to time (use $t = 0$ to represent the year 1840).

   Linear model:

   $y = -16271982.95 + 738203.4x$ $\quad r^2 = .930$

   Exponential model:

   $y = 3280877.85(1.02)^x$ $\quad r^2 = .971$

5. Which type of model – linear or exponential – seems to better fit the data pattern? Why do you think that choice is reasonable? Answers may vary.

   While neither model is a great fit, the exponential model seems more reasonable because population grows at an exponential rate and so voters should too. Also, the r^2 value for the exponential model is closer to 1.

6. In what ways is neither the linear nor the exponential model a good fit for the data pattern relating presidential election votes to time? Why do you think that modeling problem occurs?

   Neither is a very good model as neither graph falls very close to the graphed data points. Modeling these values is difficult Because there is no consistent common difference or ratio.