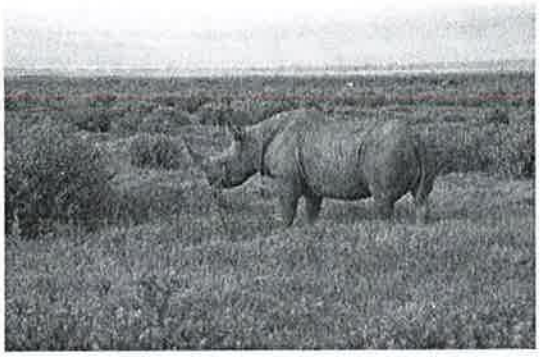
Declining Geometric Sequence Activity

Arithmetic Sequence	Common Difference
Geometric Sequence	Common Ratio
Initial Term	



Problem Situation:

The African Black Rhinoceros is the second largest of all land mammals and has been around for 40 million years. Prior to the 19th century, over 1,000,000 of the species roamed the plains of Africa; however, the number has been reduced by hunting and loss of natural habitat. The following sequence shows the population from the 1970s to early 1990s.

650,000; 195,000; 58,500; 17,550; 5265

- What are the next three terms of the sequence?

  1580, 474, 142

- How did you predict the number of rhinoceros for the 6th, 7th, and 8th terms?

  followed the pattern & multiplied by .3

- What is the initial term of the sequence?

  650 000

- What is the pattern of change?

  multiply by 0.3

- Do you think the sequence above is an arithmetic sequence? Why or why not?

  no, it is multiplying instead of adding or subtracting.

- Do you think the sequence is a growing sequence? Why or why not?

  no, the numbers are getting smaller.

Remember that an **arithmetic sequence** goes from one term to the next by always adding (or subtracting) the same value, called the **common difference**. A **geometric sequence** goes from one term to the next by always multiplying (or dividing) by the same value, called the **common**

ratio. In growing sequences the initial value and all subsequent values are multiplied by the common ratio. In declining sequences the initial value and all subsequent values are divided by the common ratio.

The number multiplied (or divided) at each stage of a geometric sequence is called the **common ratio *r***, because if you divide successive terms, you'll always get this common value. So, let's determine the common ratio *r* of the Black Rhinoceros Sequence.

650,000; 195,000; 58,500; 17,550; 5,265

195,000/650,000 = 3/10 or 0.3

58,500/195,000 = 3/10 or 0.3

17,550/58,500 = 3/10 or 0.3

5,265/17,550 = 3/10 or 0.3

The common ratio of the Black Rhinoceros is $r = 3/10$ or 0.3 and the initial term is 650,000. Recall that the initial term is simply the first term of the sequence. In our example, the initial term is 650,000. Let's now find the initial term and the common ratio of other geometric sequences.

Example 1: 4, 8/3, 16/9, 32/27, 64/81 . . .

Initial term: 4 Common ratio: $0.\overline{6}$ or $2/3$

Example 2: 6, -3, 3/2, -3/4 . . .

Initial term: 6 Common ratio: -0.5 or $-\frac{1}{2}$

Now it is time for you to determine if the following sequences are arithmetic or geometric. If the sequences are geometric, then determine if they are growing or declining. On the next page, you will find some problems to help you practice your skills on sequences.

Guided Practice with Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1) 56, 28, 14, 7,...

yes 0.5

2) 64, -48, 36, -27,...

yes -0.75

3) 9, 6, 3, 0, -3, -6, ...

no

4) 1000, 100, 10, . . .

yes 0.1

5) 8, 2, ½, . . .

yes 0.25

6) 18, 6, 2, . . .

yes $0.\overline{3}$

Given the initial term and common ratio, write the first 6 terms of the sequence.

7) $a_1 = 7, r = 2/3$ 7, 14/3, 28/9, 56/27, 112/81, 224/243

8) $a_1 = 5, r = ½$ 5, 5/2, 5/4, 5/8, 5/16, 5/32

9) $a_1 = 3, r = 3/5$ 3, 9/5, 27/25, 81/125, 243/625, 729/3125

10) $a_1 = 3/7, r = ¼$ 3/7, 3/28, 3/112, 3/448, 3/1792, 3/7168

Problem Situation:

A hot vanilla latte from McDonalds is poured into a cup and allowed to cool while you are riding to school. The difference between the latte temperature and room temperature is recorded every minute for 10 minutes. The sequence is found below:

80, 72, 65, 58, 52, 47, 43, 38, 34, 31, 28

11) Is this sequence geometric? If so, what is the approximate common ratio?

geometric 0.9

12) How is problem similar or different to the Black Rhinoceros problem in the lesson?

The pattern is a decreasing geometric sequence also. However, the initial values and common ratios are different.

Independent Practice with Sequences – Day 87

Are the following sequences arithmetic, geometric, or neither? If they are arithmetic, state the value of d. If they are geometric, state r.

1. 6, 12, 18, 24, ... arithmetic $d = 6$
2. 6, 11, 17, ... neither
3. 2, 14, 98, 686, ... geometric $r = 7$
4. 160, 80, 40, 20, ... geometric $r = 0.5$
5. -40, -25, -10, 5, ... arithmetic $d = 15$
6. 7, -21, 63, -189, ... geometric $r = -3$
7. 2/3, (2/3)2, (2/3)3, ... geometric $r = 2$
8. 1/3, 4/3, 7/3, 10/3,... arithmetic $d = 1$
9. 10, 10/8, 10/64, ... geometric $r = 1/8$
10. 10, 80, 640, 5120, ... geometric $r = 8$
11. 1/3, 8/3, 64/3, 512/3, ... geometric $r = 8$
12. Which of the geometric sequences are growing? 3, 7, 10, 11
13. Which of the geometric sequences are declining? 4, 9

Real life application:

14. You throw a SuperBall on the cement as hard as you can and watch it bounce until it stops. You notice the first bounce reaches a height of 200 ft, but the second bounce reaches only half of that height. How high will the 7th bounce reach?

    200, 100, 50, 25, 12.5, 6.25, 3.125 feet

    a. What type of sequence is illustrated by this problem?

    geometric $r = \frac{1}{2}$

    b. Is this sequence growing or declining?

    declining