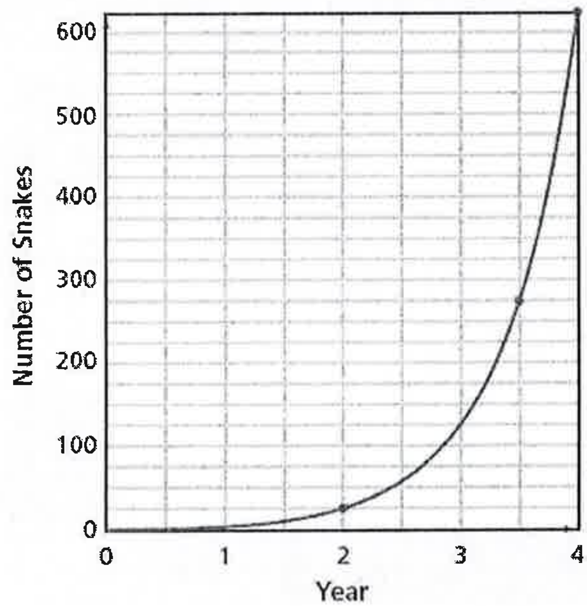
From Geometric Sequences to Tables and Graphs

The geometric sequence from the Brown Tree Snake problem (1, 5, 25, 125, 625 . . .) can be written in the form of a table, as shown below:

Year	0	1	2	3	4
# of Snakes	1	5	25	125	625

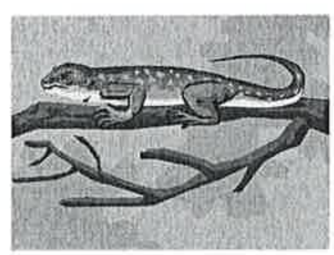
The Brown Tree Snake was first introduced to Guam in year 0. At the end of year 1, five snakes were found; at the end of year 2, twenty-five snakes were discovered, and so on. Since we now have a table of the information, a graph can be drawn, where the year is the independent variable (x) and the number of snakes is the dependent variable (y). See below:

Notice that the graph of the table is not a straight line. Therefore, the graph is not linear in nature, which we already from the fact that the sequence is not arithmetic. Rather, the graph is curved and moves in a growing fashion very rapidly due to the fact that the common ratio *r* of this sequence is 5. The curved graph of this problem situation is known as an **exponential growth** function. An exponential growth function occurs when the common ratio *r* is greater than one. Tables and graphs make viewing the data from the problem situation easier to see and we can easily see from either the table or graph that in year 3, the snake population is 125.



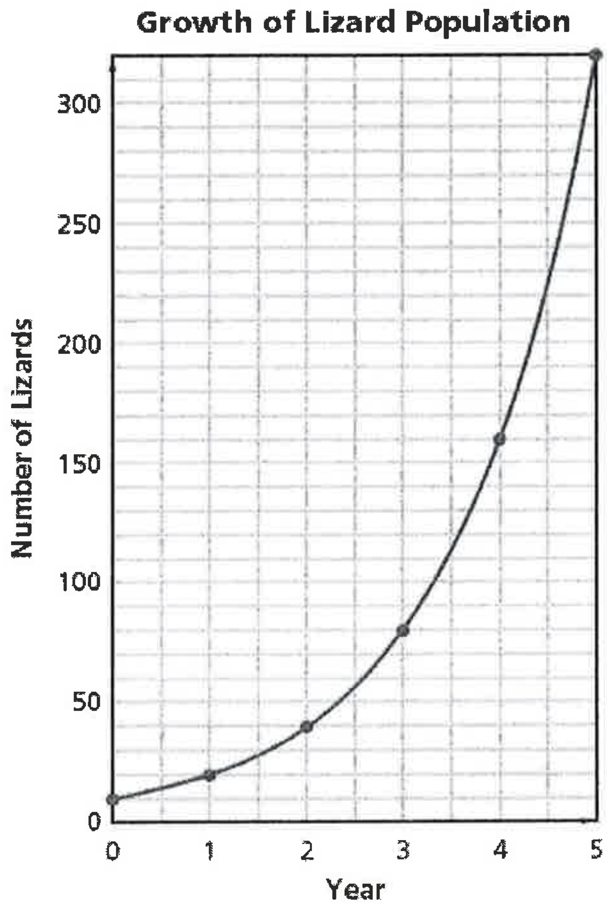
Let us look at a similar population growth for a certain kind of lizard in both a table and graph. Use either one or both to answer the questions below.

Year	0	1	2	3	4	5
Number of lizards	10	20	40	80	160	320



$\frac{20}{10} = 2$ $\frac{320}{160} = 2$

common ratio = 2



Notice from the shape of the graph that the information is exponential in nature.

1. What information does the point (2, 40) on the graph tell you?
40 lizards in 2 years
2. What information does the point (1, 20) on the graph tell you?
20 lizards in 1 year
3. When will the population exceed 100 lizards? 3.32 years

4. Explain how to find the common ratio, using either the table or graph.
divide any y-value by the previous one
5. If the information from the table were written as a sequence, what is the initial term? 10

6. How could we find the 10th term in the table, graph, or sequence?
Keep multiplying by 2 until the 10th time

Discuss your answers with your group or with a partner. Now let us explore some more problems.

The Mice Problem

A population of mice has a growth factor (otherwise known as the common ratio) of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.

1. How many mice were in the population initially (at 0 months)? Explain how you found this number. 12 mice (divide 36 by 3)

2. Write a sequence to show how the mice population is growing.
12, 36, 108, 324, 972
3. Is this sequence arithmetic or geometric? Explain how you know.
geometric, the values are being multiplied by 3.
4. Now, put your sequence into the table below.

Months	0	1	2	3
Number of Mice	12	36	108	324

5. Is the graph of the table going to be a straight line or a curve? Explain your answer.

a curve. You can see it on the graph.
Also, common ratios in geometric series generate curved graphs.

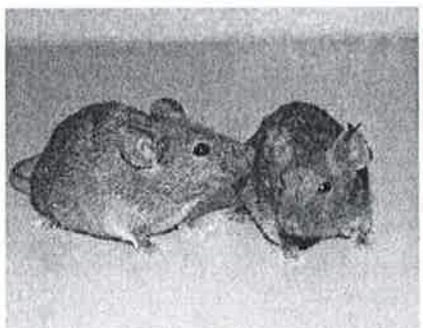
6. Graph the table to make sure of your answer on the graph below. Make sure you label and title the graph below.

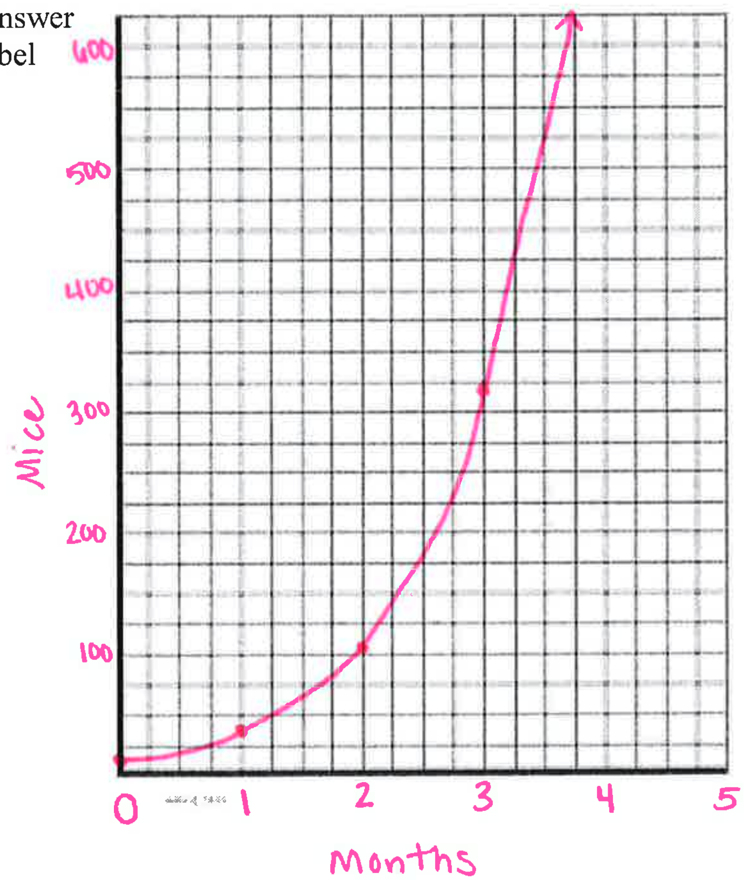
   a. What is your scale for the x-axis?

   0.25

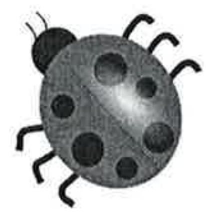
   b. What is your scale for the y-axis?

   20





The Ladybug Invasion



As a biology project, Tamara is studying the growth of a ladybug population. She starts her experiment with 5 ladybugs. The next month she counts 15 ladybugs.

1. Suppose the ladybug population is growing arithmetically. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

   2 mos. 3 mos. 4 mos.

   5, 15, 25, 35, 45, 55

2. What is the common difference?

   10

3. Now put the sequence into a table in the space below.

Months	0	1	2	3	4	5
Ladybugs	5	15	25	35	45	55

4. How long will it take the ladybug population to reach 200 if it is growing linearly?

   19.5 months

5. Suppose the ladybug population is growing exponentially. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

   5, 15, 45, 135, 405

   2 mos. 3 mos. 4 mos.

6. What is the common ratio?

3

7. Now put the sequence into a table in the space below.

months	0	1	2	3	4	5
ladybugs	5	15	45	135	405	1215

8. How long will it take the ladybug population to reach 200 if it is growing exponentially?

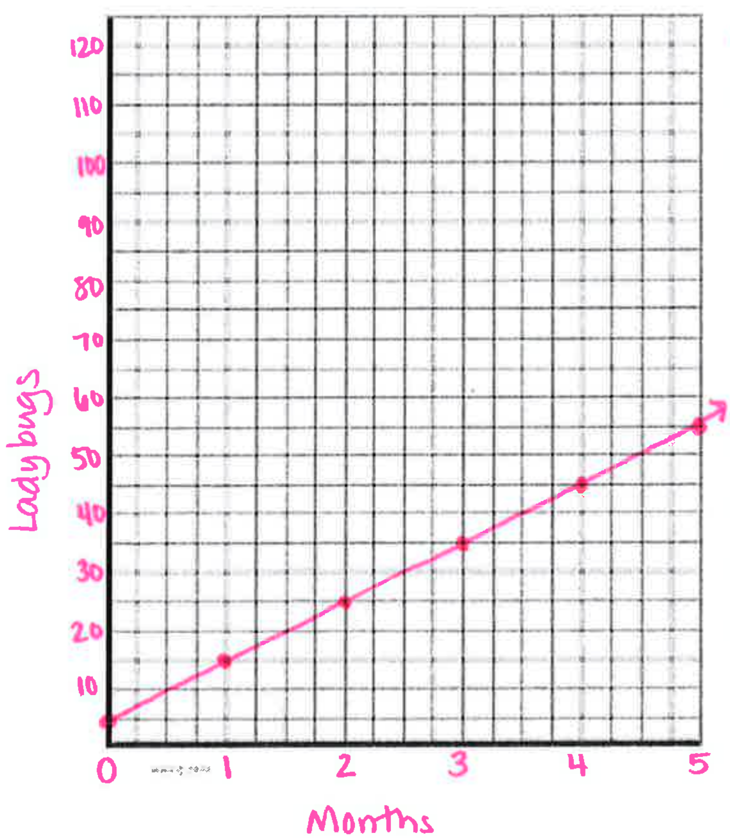
3.35 months

9. Graph both tables on the designated graphs below. Be sure to label your axes.

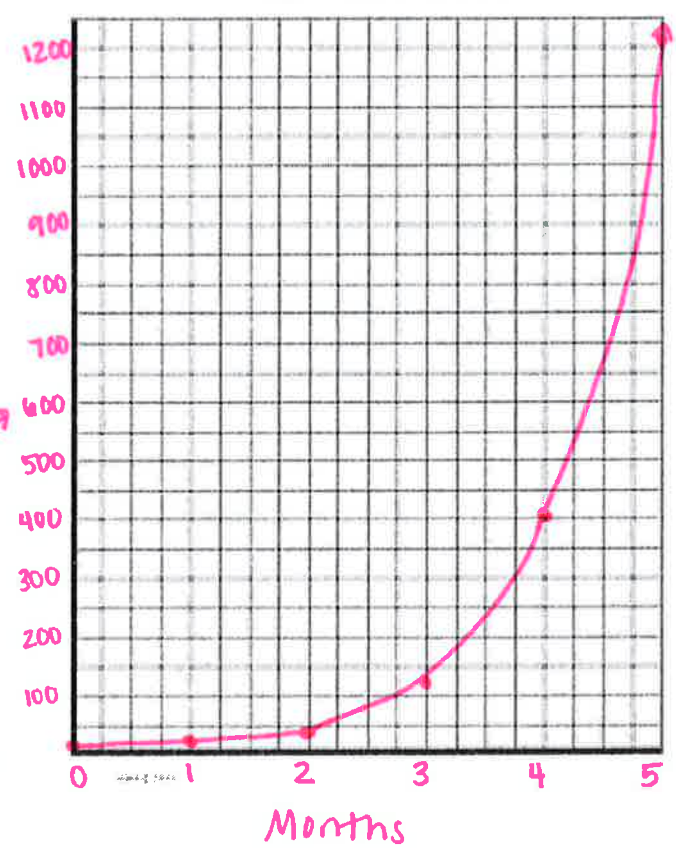
see below

10. Why does it take the ladybug population longer to reach 200 when it grows linearly?

Linear Growth

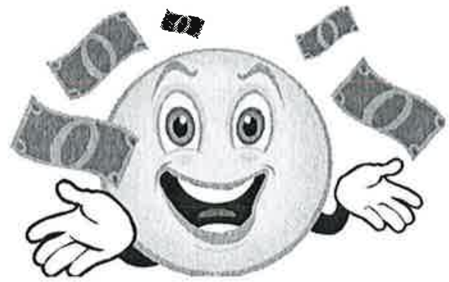


Exponential Growth



Adapted from *Growing, Growing, Growing, Exponential Relationships*, Connected Mathematics 2, Pearson, 2009.

Who Wants to Be Rich?



Students at a local school want to have a quiz show called *Who Wants to Be Rich?* Contestants will be asked a series of questions. A contestant will play until he or she misses a question. The total prize money will grow with each question answered correctly.

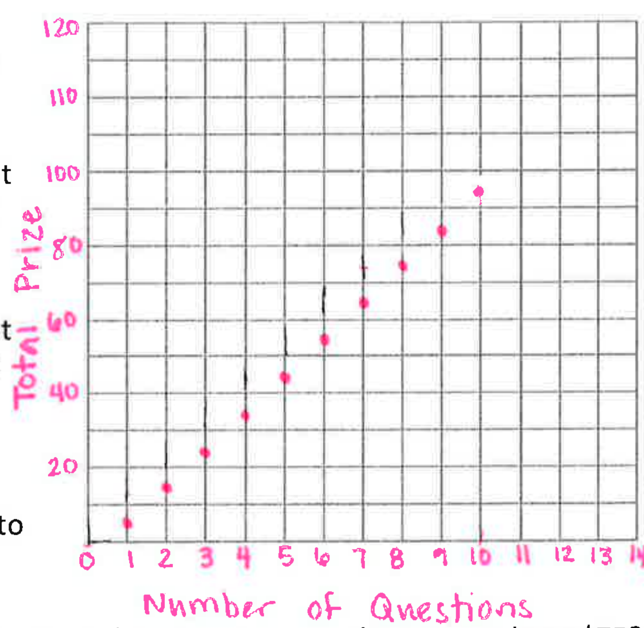
Lucy and Pedro are on the prize winnings committee and have different view of how prize winning should be awarded. Their plans are outlined below for your consideration. Review them by answering the questions following the plans. Remember that the committee has a fixed amount of money to use for this quiz show.

1. Lucy proposes that a contestant receives $5 for answering the first question correctly. For each additional correct answer, the total prize would increase by $10.
   a. For Lucy's proposal, complete the table below.

Number of questions	1	2	3	4	5	6	7	8	9	10
Total prize	5	15	25	35	45	55	65	75	85	95

   b. Sketch the graph of correctly answered questions 1-10. Be sure to title your graph and label the axes.

   ✓



* Mention that this data is discrete, not continuous.

   c. How much money would a contestant win if he or she correctly answered 6 questions?

   $55

   d. How much money would a contestant win if he or she correctly answered 9 questions?

   $85

   e. How many questions would a contestant need to answer correctly to win at least $50?

   6

   f. How many questions would a contestant need to answer correctly to win at least $75?

   8

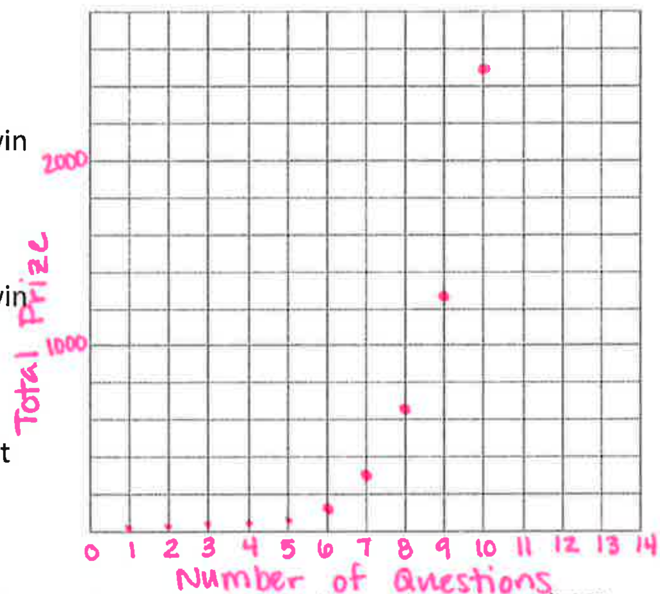
   g. How is this table growing? Is this a linear or exponential growth pattern?

   linear

2. Pedro also proposes that the first question should be worth $5. However, he thinks a contestant's winnings should double with each subsequent answer.
   a. For Pedro's proposal, complete the table below.

Number of questions	1	2	3	4	5	6	7	8	9	10
Total prize	5	10	20	40	80	160	320	640	1280	2560

   b. Sketch the graph of correctly answered questions 1-10.
   ✓



   c. How much money would a contestant win if he or she correctly answered 6 questions?
   $160

   d. How much money would a contestant win if he or she correctly answered 9 questions?
   $1280

   e. How many questions would a contestant need to answer correctly to win at least $50?
   5

   f. How many questions would a contestant need to answer correctly to win at least $75?
   5

   g. How is this table growing? Is this a linear or exponential growth pattern?
   exponential

3. Which plan is better for the contestants? Explain your reasoning.
   It depends on how many questions they get correct
   1, 2, or 3 questions → Lucy's plan pays more
   4-10 questions → Pedro's plan pays more

4. Which plan is better for the school? Explain your reasoning.
   Lucy's plan is better for the school because it costs them less.

Adapted from Growing, Growing, Growing Exponential Relationships, Connected Mathematics 2, Pearson Publishing, 2009.

Independent Practice: Charity Donations

Mari's wealthy Great-aunt Sue wants to donate money to Mari's school for new computers. She suggests three possible pans for her donations.

Plan 1: Great-aunt Sue's first plan is give money in the following way: 1, 2, 4, 8, ... She will continue the pattern in this table until day 12. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Donation	$1	$2	$4	$8	$16	32	64	128	256	512	1024	2048

Total
$4095

Plan 2: Great-aunt Sue's second plan is to give funds in the following way: 1, 3, 9, 27, ... She will continue the pattern in this table until day 8. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7	8
Donation	$1	$3	$9	$27	81	243	729	2187

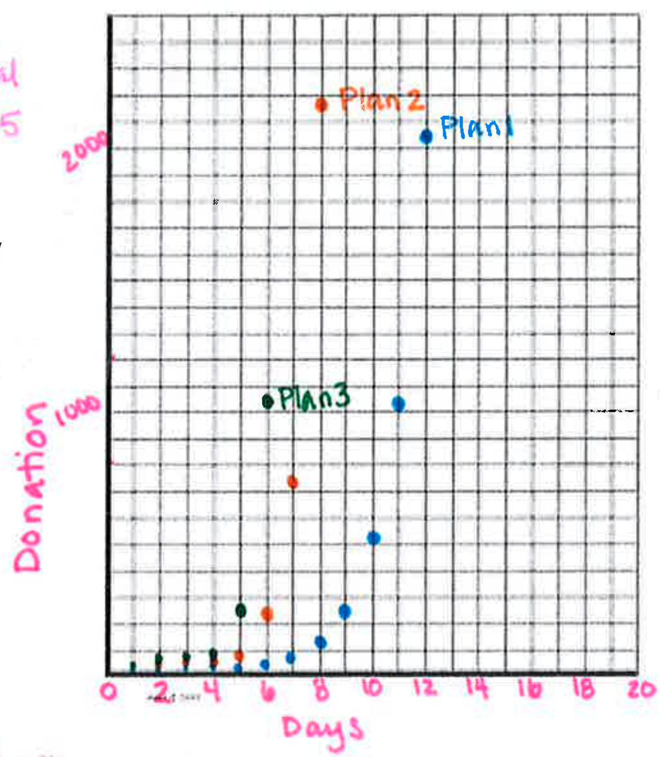
Total
$3280

Plan 3: Great-aunt Sue's third plan is to give money in the following way: 1, 4, 16, 64, . . . She will continue the pattern in this table until day 6. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6
Donation	$1	$4	$16	$64	256	1024

Total
$1365

Graph each plan on the same graph to the right.



1. How much does each plan give the school on day 6?

   ① $32 ② $243 ③ $1024

2. What is the common ratio (growth rate) for each plan?

   a. Plan 1 2

   b. Plan 2 3

   c. Plan 3 4

3. Which plan should the school choose? Why?

   Plan 1 – they earn more money.

4. Which plan will give the school the **greatest total** amount of money?

   Plan 1

Jason is planning to swim in a charity swim-a-thon. Several relatives have agreed to sponsor him in this charity event. Each of their donations is explained below.

Grandfather: I will give you $1 for the first 1 lap, $3 for the 2nd lap, $5 for the 3rd lap, $7 for the 4th lap, and so on.

Father: I will give you $1 for the first lap, $1.50 for the 2nd lap, $2.25 for the 3rd lap, $3.38 for the 4th lap, and so on.

Aunt June: I will give you $2 for the first lap, $3.50 for the 2nd lap, $5 for the 3rd lap, $6.50 for the 4th lap, and so on.

Uncle Bob: I will give you $1 for the first lap, $1.25 for the 2nd lap, $1.56 for the 3rd lap, $1.95 for the 4th lap, and so on.

5. Decide whether each donation sequence is exponential, linear, or neither.

   a. Grandfather's Plan linear (+ $2)

   b. Father's Plan exponential (x 1.5)

   c. Aunt June's Plan linear (+ $1.50)

   d. Uncle Bob's Plan exponential (x 1.25)

6. Complete the table for each sequence below.

Grandfather's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10	Total
Donation	$1	$3	$5	$7	9	11	13	15	17	19	$100

Father's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10	Total
Donation	$1	$1.50	$2.25	$3.38	5.06	7.59	11.39	17.09	25.63	38.44	$113.33

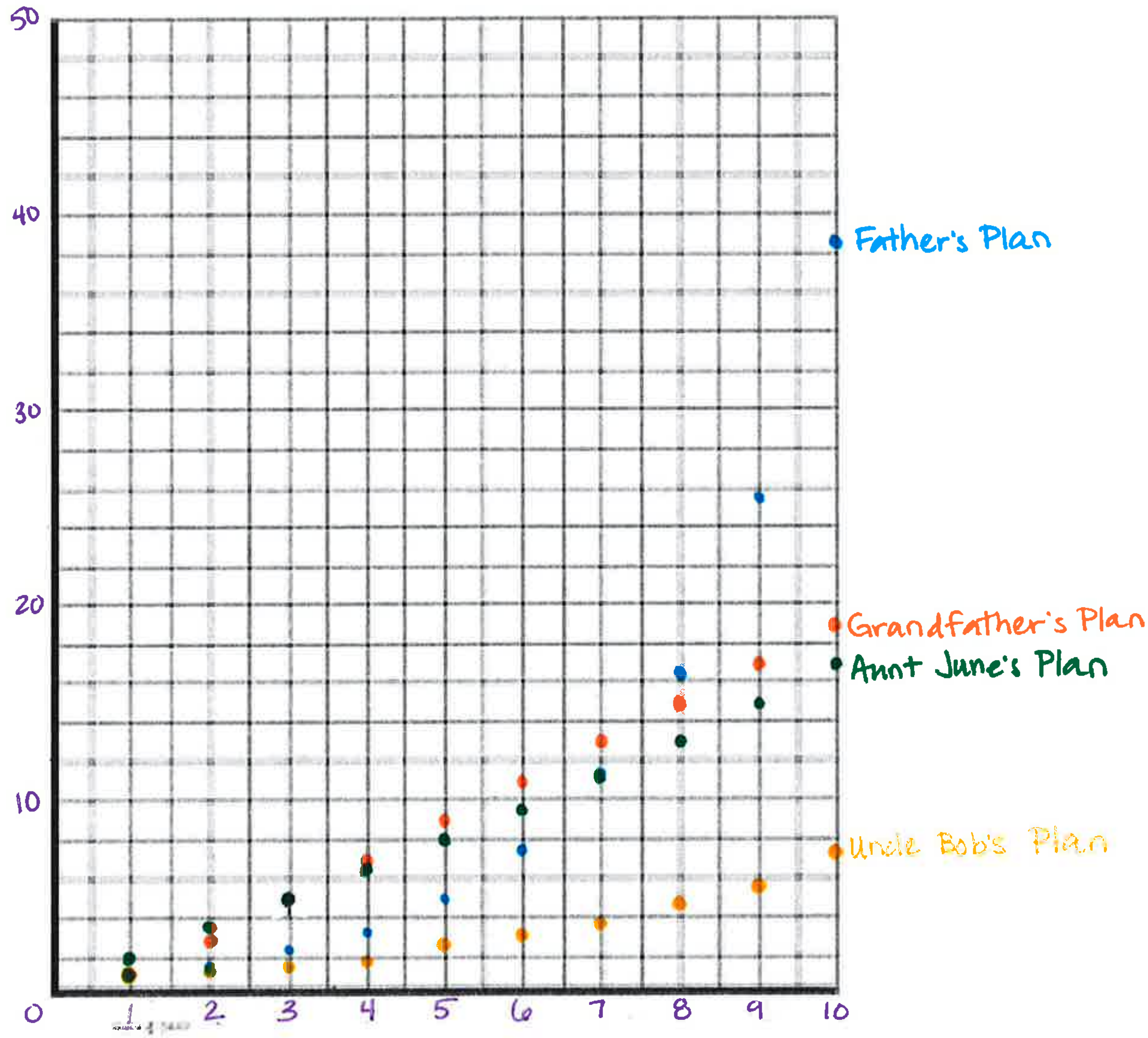
Aunt June's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10	Total
Donation	$2	$3.50	$5	$6.50	8.00	9.50	11.00	12.50	14.00	15.50	$87.50

Uncle Bob's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10	Total
Donation	$1	$1.25	$1.56	$1.95	2.44	3.05	3.81	4.77	5.96	7.45	$33.24

7. Graph each table on the graph below. Label each line or curve. Title the graph and label the axes.



8. Use either the table or graph to determine the **total money** Jason will raise for each plan if he swims 10 laps.

a. Grandfather's Plan $100

b. Father's Plan $113.33

c. Aunt June's Plan $87.50

d. Uncle Bob's Plan $33.24

Adapted from *Growing, Growing, Growing, Exponential Relationships*, Connected Mathematics 2, Pearson, 2009.