## Bouncing Ball Investigation

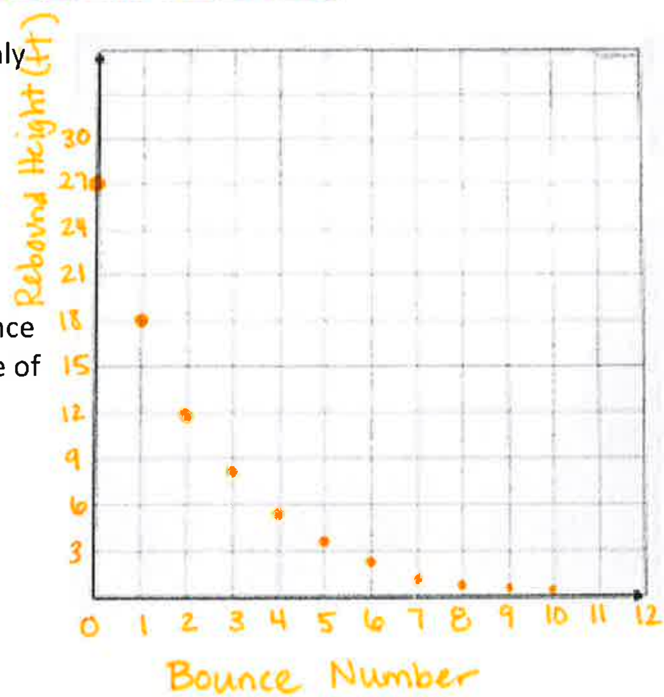
The majority of sports in America revolve around some sort of ball. One of the most important aspects of those balls is the elasticity or bounciness of the ball. For example, when a new golf ball is dropped onto a hard surface, it rebounds to about 2/3 of the height it was dropped. The pattern of bounces resembles the declining pattern of a geometric sequence.

**Example:** Suppose a new golf ball is dropped from a height of 27 feet onto a parking lot and keeps bouncing up and down. Each bounce is 2/3 of the drop height as shown below:

27, 18, 12, 8, . . .

We can take this geometric sequence and put it into a table to keep up with the number of times the golf ball bounces. Complete the rest of the table below and graph on the coordinate plane. Then answer the questions found below the graph.

| **Bounce Number** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Rebound Height (in feet)** | 27 | 18 | 12 | 8 | 5.3 | 3.5 | 2.4 | 1.6 | 1.1 | .7 | .5 |




1. Why does the "practical domain" for this problem only include positive whole numbers?

   because we are counting bounces and you can't have negative or partial bounces.

2. How does the rebound height change from one bounce to the next? How is that pattern shown by the shape of the data plot?

   The rebound height is 2/3 of the previous height, so each new point has 2/3 the y-value of the previous point.

3. What is the initial value of the data?

   27 feet

4. What is the common ratio?

   $2/3$

5. What rule relating NOW and NEXT shows how to calculate the rebound height for any bounce from the height of the preceding bounce?

   NEXT = NOW(2/3), start at 27

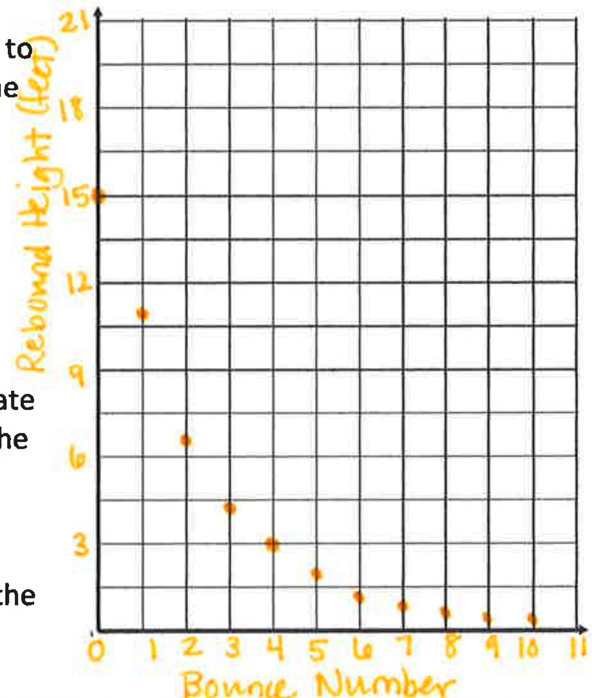
6. What rule beginning with "y = " show how to calculate the rebound height after any number of bounces?

   $y = 27(2/3)^x$

The table, graph and equations illustrate the concept of **exponential decay**. Exponential decay occurs when the common ratio $r$ falls between 0 and 1 ($0 < r < 1$). The graphs of exponential decay problems illustrate a rapid decline of the data.

**Your Turn:**
How will the data table, plot, and rules for calculating rebound height change if the ball drops first from only 15 feet? Repeat the problem above, this time using a drop height of 15 feet.

| **Bounce Number** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Rebound Height (in feet)** | 15 | 10 | $6.\overline{6}$ | $4.\overline{4}$ | 3.0 | 2.0 | 1.3 | .9 | .6 | .4 | .3 |




1. How does the rebound height change from one bounce to the next? How is that pattern shown by the shape of the data plot?

   The rebound height is still 2/3 of the previous height, so the new points have a y-value that is 2/3 that of the previous point.

2. What rule relating NOW and NEXT shows how to calculate the rebound height for any bounce from the height of the preceding bounce?

   $NEXT = NOW\left(\frac{2}{3}\right)$ start at 15

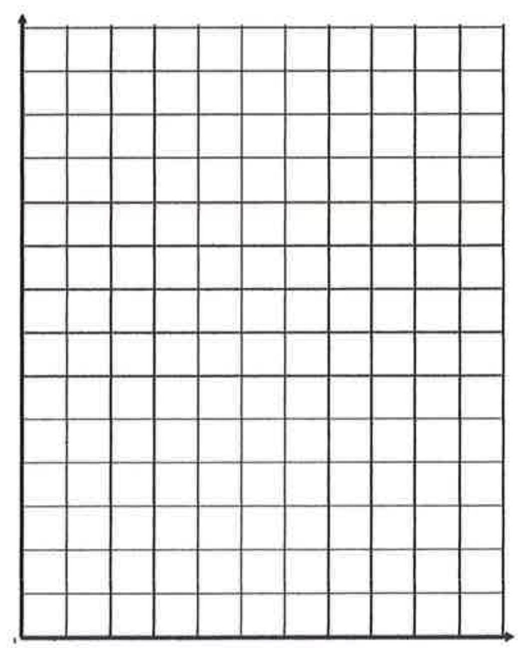
3. What rule beginning with "y = " show how to calculate the rebound height after any number of bounces?

   $y = 15\left(\frac{2}{3}\right)^x$

Data from actual tests of golf balls will not exactly match the predictions made in the above predictions from the recursive NOW-NEXT and explicit "y =" equations. You will simulate the kind of quality control testing that is done by manufacturing factories by running some experiments in your classroom. Work with a group of three or four people to complete the next problem.

**Problem 1:** Get a golf ball and tape measure or meter stick from your teacher to use in your group. Decide on a method for measuring the height of successive rebounds after the ball is dropped from a height of at least 8 feet. Collect data on the rebound height for successive bounces of the ball in the table below. Repeat the same experiment 2 times and then average your 2 sets of data. Graph the TOP and BOTTOM rows of data on the coordinate plane.

answers will vary



| **Bounce Number** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Trial 1: Rebound Height (in feet)** | 8 | | | | | |
| **Trial 2: Rebound Height (in feet)** | | | | | | |
| **Average Rebound Height (in feet)** | | | | | | |

Lesson adapted from Core-Plus Mathematics, Glencoe McGraw-Hill, 2008.

1. Compare the pattern of your data to that of the predicted data at the top of the first page. Would a rebound height factor other than 2/3 give a better model for your data? Explain.

2. Write a rule using NOW-NEXT that relates the rebound height of any bounce of your tested ball to the height of the preceding bounce.

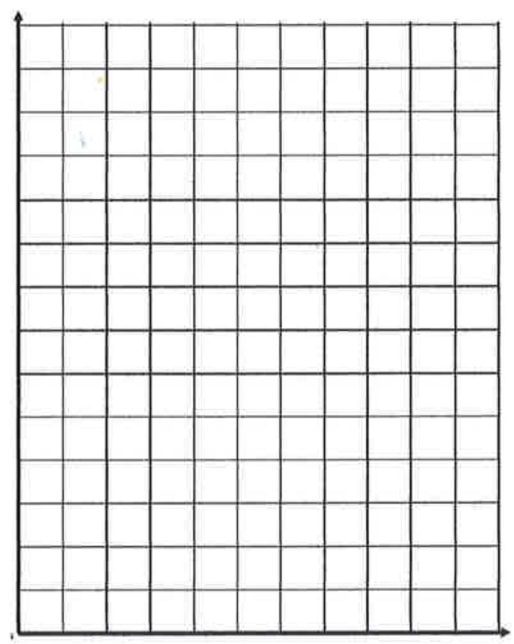
3. Write a "y =" rule to predict the rebound height after any bounce.

4. How close were your equations to accurately representing your data?

**Problem 2:** Repeat the experiment of Problem 1 with some other ball such as a tennis ball or volleyball. Use the table to record the data collected and the coordinate plane to graph the data.

Type of Ball used:

| **Bounce Number** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Trial 1: Rebound Height (in feet)** | 8 | | | | | |
| **Trial 2: Rebound Height (in feet)** | | | | | | |
| **Average Rebound Height (in feet)** | | | | | | |



1. What is a reasonable rebound height for your data?

2. Use your rebound height to write a rule using NOW-NEXT that relates the rebound height of any bounce of your tested ball to the height of the preceding bounce.

3. Write a "y =" rule to predict the rebound height after any bounce.

**Activity Reflection:**

1. What similarities do you find with each problem? What differences?

   All were declining sequences (exponential decay), but they had different $a_1$ & r values

2. What do the tables, graphs, and rules in these problems have in common with exponential growth problems that you studied earlier in the unit?

   They all have initial values that correspond with the y-intercept and common ratios, although these common ratios are between 0 and 1, not more than 1.

3. What is the general NOW-NEXT equation to be used with exponential decay problems?

   NEXT = NOW(r), start at $a_1$

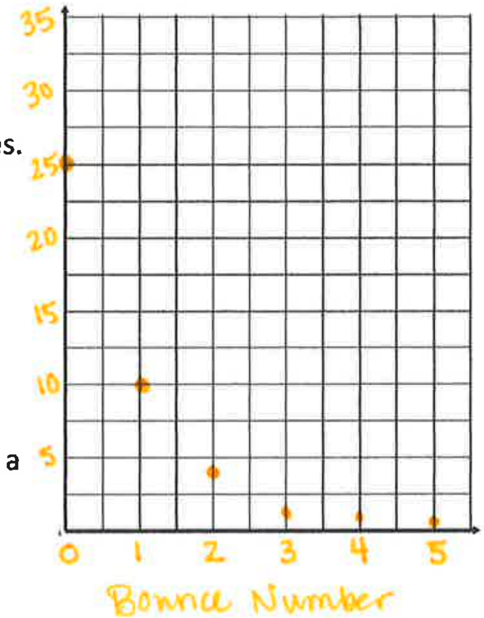
4. What is the general "y = " equation to be used with exponential decay problems?

   $y = a_1(r)^x$ $a_1$ = initial value r = common ratio

## Independent Practice with Bouncing Balls Exponential Decay Problems

1) When dropped on to a hard surface, a brand new softball should rebound to about 2/5 the height from which it is dropped.
   a. If the softball is dropped 25 feet from a window onto concrete, what pattern of rebound heights can be expected?

   Each rebound height will be 2/5 of the previous height.




   b. Make a table and plot of predicted rebound data for 5 bounces.

| Bounce Number | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rebound Height (in feet) | 25 | 10 | 4 | 1.6 | .64 | .256 |

   c. What NOW-NEXT rule and "y = " rules give ways of predicting rebound height after any bounce?

   NEXT = NOW(2/5) start at 25 $\quad y = 25\left(\frac{2}{5}\right)^x$

2) Here are some data from bounce tests of a softball dropped from a height of 10 feet.

| Bounce Number | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rebound Height (in feet) | 10 | 3.8 | 1.3 | 0.6 | 0.2 | 0.05 |

   a. What do these data tell you about the quality of the tested softball?

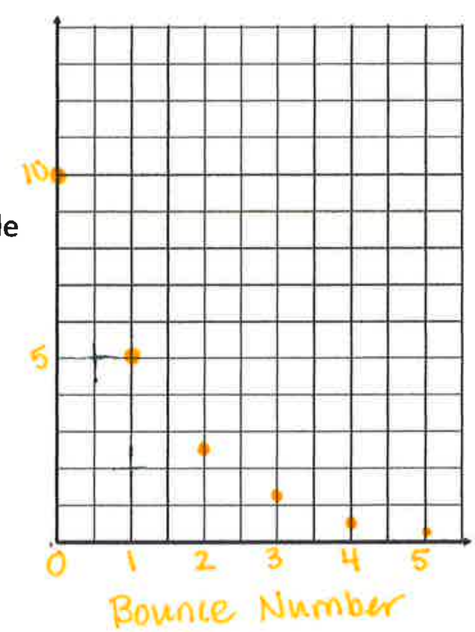
   A new ball rebounds to 2/5 or 0.4 times the original height. Since this one only rebounds to 0.38 times the original height, it must be older.

   b. What are the first six bounce heights would you expect from this ball if it were dropped from 20 feet instead of 10 feet?

   20, 7.6, 2.888, 1.097, 0.417, 0.158

3) If a basketball is properly inflated, it should rebound to about ½ the height from which it is dropped.
   a. Make a table and plot showing the pattern to be expected in the first 5 bounces after a ball is dropped from a height of 10 feet.

| Bounce Number | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rebound Height (in feet) | 10 | 5 | 2.5 | 1.25 | .625 | .313 |




   b. At which bounce will the ball first rebound less than 1 foot? Show how the answer to this question can be found in the table and on the graph.

   The fourth bounce is less than one foot. It is the first y-value below 1 on the table and graph.

   c. Write a rule using NOW-NEXT and a rule beginning "y = " that can be used to calculate the rebound height after many bounces.

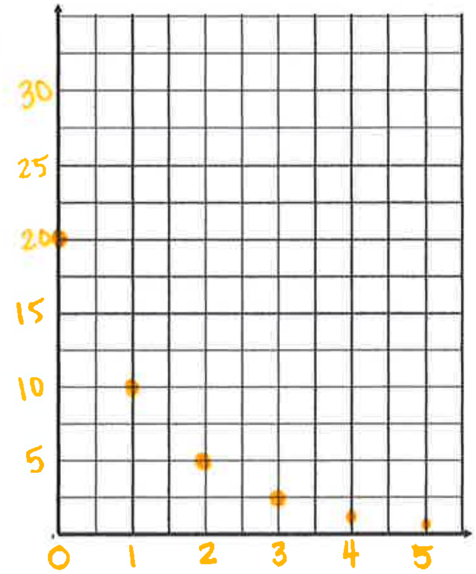
   NEXT = NOW$\left(\frac{1}{2}\right)$ start at 10 $\quad y = 10\left(\frac{1}{2}\right)^x$

d. How will the data table, plot, and rules change for predicting rebound height if the ball is dropped from a height of 20 feet?

| Bounce Number | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rebound Height (in feet) | 20 | 10 | 5 | 2.5 | 1.25 | .625 |

NOW-NEXT Rule: NEXT = NOW(1/2), start at 20

Y = $20(1/2)^x$




e. How will the data table and rules change for predicting rebound height if the ball is somewhat over-inflated and rebounds to 3/5 of the height from which it is dropped?

| Bounce Number | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rebound Height (in feet) | 20 | 12 | 7.2 | 4.3 | 2.6 | 1.6 |

NOW-NEXT Rule: NEXT = NOW(3/5)
start at 20

Y = $20(3/5)^x$

Adapted from Core-Plus Mathematics, Glencoe McGraw-Hill, 2008.