

One Grain of Rice

NAME _____

Common Ratio

NOW-NEXT

In the book *One Grain of Rice* by Demi, the main character Rani cleverly tricks the raja into giving rice to the village. Use the story from the book to answer the questions below.

1. Estimate how many grains of rice you think Rani will have at the end of 30 days.
2. Use the chart below to record the number of grains of rice Rani would receive each day.

Day 1 1 grain of rice	Day 2 2 grain of rice	Day 3 4 grain of rice	Day 4 8 grain of rice	Day 5 16 grain of rice	Total After 5 Days 31
Day 6 32 grain of rice	Day 7 64 grain of rice	Day 8 128 grain of rice	Day 9 256 grain of rice	Day 10 512 grain of rice	Total After 10 Days 1023
Day 11 1024 grain of rice	Day 12 2048 grain of rice	Day 13 4096 grain of rice	Day 14 8192 grain of rice	Day 15 16384 grain of rice	Total After 15 Days 32767
Day 16 32768 grain of rice	Day 17 65536 grain of rice	Day 18 131072 grain of rice	Day 19 262144 grain of rice	Day 20 524288 grain of rice	Total After 20 Days 1048575
Day 21 1048576 grain of rice	Day 22 2097152 grain of rice	Day 23 4194304 grain of rice	Day 24 8388608 grain of rice	Day 25 16777216 grain of rice	Total After 25 Days 33554431
Day 26 33554432 grain of rice	Day 27 67108864 grain of rice	Day 28 134217728 grain of rice	Day 29 268435456 grain of rice	Day 30 536870912 grain of rice	Total After 30 Days 1074741823

3. If the story continued and you know how many grains of rice Rani receives on Day 30, how can you determine how many grains of rice she would receive on Day 31?

multiply 536870912 by 2

4. How can you determine how many grains of rice she would receive on Day 35?

continue the pattern $536870912 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

5. How can you determine how many grains of rice she would receive on Day 40?

$536870912 \cdot 2^{10}$

6. If you know how many grains of rice she receives on a certain day, how can you determine how many grains of rice she will receive 2 days later? . . . 10 days later?

multiply by 2 for each day later

for 2 days \rightarrow starting amount $\cdot 2^2$

for 10 days \rightarrow starting amount $\cdot 2^{10}$

7. Write a sentence that describes how many grains of rice Rani receives each day.

Rani receives twice as much as the day before.

8. What number is being used to advance the pattern? Is this a common difference or a common ratio?

you are multiplying by 2, so 2 is the common ratio

9. Use the words *NOW* and *NEXT* to write a rule to express the pattern. The **NOW-NEXT form** is the recursive process of getting from one number to the next number in the sequence.

$$\text{NEXT} = \text{NOW} \cdot 2$$

10. Use the pattern you discovered to write an explicit equation for the rice acquired each day. (number of day, rice received)?

$$y = 2^{x-1} \quad x = \text{number of days} \quad y = \text{rice received}$$

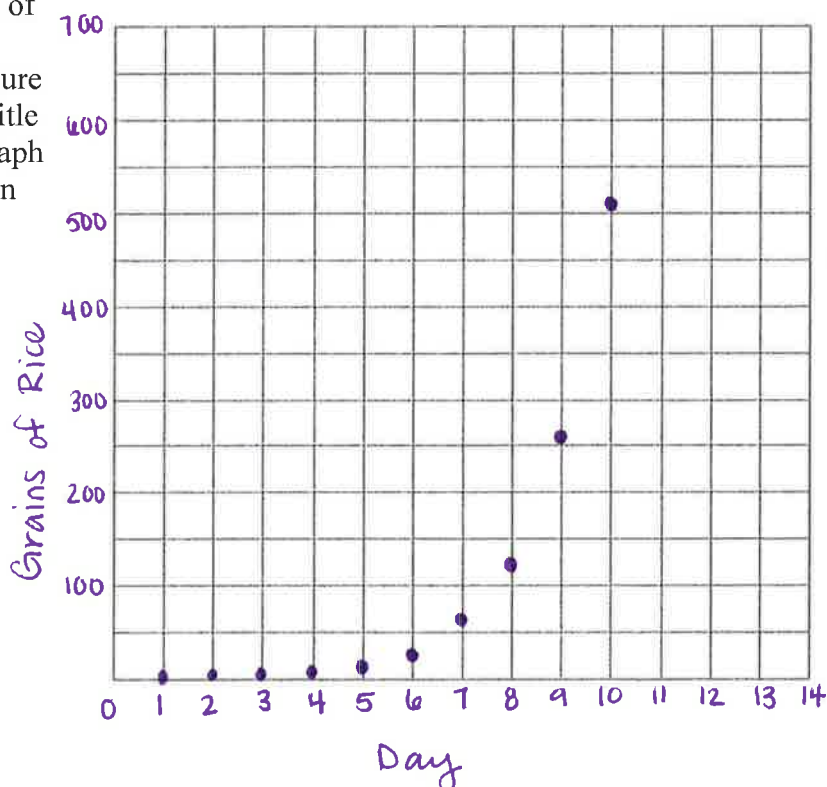
11. Write a sentence that describes the total number of grains of rice Rani will receive through a certain number of days.

One less than what she'd get the next day in the pattern.

12. Test each of your equations to see if they generate the values in the table. Were your algebraic equations correct? If not, modify your equations and test them until you are certain they are correct. Record the changes you make so that you can explain to others how you arrived at your final equations.

13. Graph the first ten days of the rice reward on the graph to the right. Be sure to label your axes and title your graph. Is this a graph of a linear function or an exponential function?

no!



Pay It Forward, Again

At the beginning of this unit we examined the *Pay It Forward* class project that Trevor McKinney came up with. Let us revisit this situation and take a deeper look at what transpired.

1. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the *Pay It Forward* process.

Stage of Process	0	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	1	3	9	27	81	243	729	2187	6561	19683	59049

2. Then plot the data on a graph. Make sure you have accurate axis labels and scales.

discrete graph

3. What pattern do you notice in the number of good deeds?

multiply by 3
each time

4. What is the common ratio?

3

5. How is that pattern change shown in the plot of the data?

each point is 3 times as
high as the previous point

6. How many stages of the *Pay It Forward* process will be needed before a total of at least 20,000 good deeds will be done?

10

7. Write a NOW-NEXT rule to illustrate the *Pay It Forward* process.

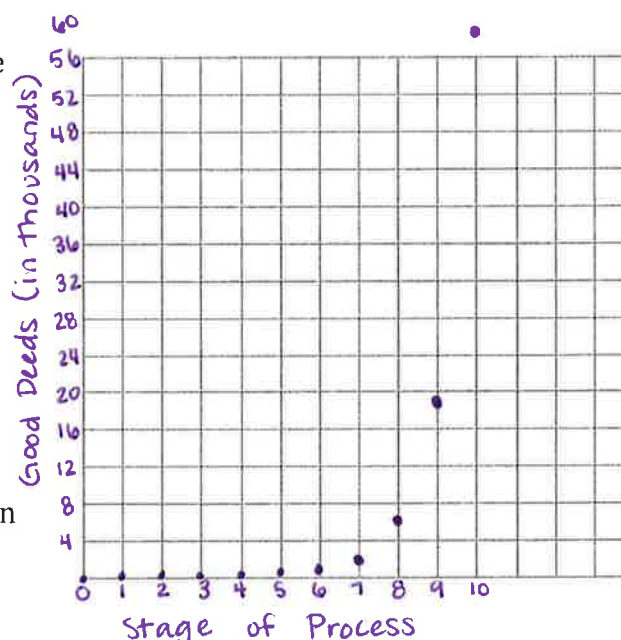
$NEXT = NOW \cdot 3$

8. Write an NOW-NEXT rule that would show the number of good deeds at a stage number if each person in the process does good deeds for **two** others.

$NEXT = NOW \cdot 2$

9. How would the NOW-NEXT rule change if each person in the process does good deed for **four** other people?

$NEXT = NOW \cdot 4$



The Million Dollar Mission

Name _____

You're sitting in math class, minding your own business, when in walks a Bill Gates kind of guy - the real success story of your school. He's made it big, and now he has a job offer for you. He doesn't give too many details, mumbles something about the possibility of danger. He's going to need you for 30 days, and you'll have to miss school. (Won't that just be too awful?) And you've got to make sure your passport is current. (Get real, Bill, this isn't Paris). But do you ever sit up at the next thing he says:

You'll have your choice of two payment options:

1. One cent on the first day, two cents on the second day, and double your salary every day thereafter for the thirty days; or
2. Exactly \$1,000,000. (That's one million dollars!)

You jump up out of your seat at that. You've got your man, Bill, right here. You'll take that million. You are there. And off you go on this dangerous million-dollar mission.

So how smart was this guy? Did you make the best choice? Before we decide for sure, let's investigate the first payment option. Complete the table for the first week's work.

First Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
1	.01	.01
2	.02	.03
3	.04	.07
4	.08	.15
5	.16	.31
6	.32	.63
7	.64	1.27

So, after a whole week you would have only made \$1.27.

That's pretty awful, all right. There's no way to make a million in a month at this rate. Right? Let's check out the second week. Complete the second table.

Second Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
8	1.28	2.55
9	2.56	5.11
10	5.12	10.23
11	10.24	20.47
12	20.48	40.95
13	40.96	81.91
14	81.92	163.83

Well, you would make a little more the second week; at least you would have made \$1163.83. But there's still a big difference between this salary and \$1,000,000. What about the third week?

Third Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
15	1163.84	327.67
16	327.68	655.35
17	655.36	1310.71
18	1310.72	2621.43
19	2621.44	5242.87
20	5242.88	10485.75
21	10485.76	20971.51

We're getting into some serious money here now, but still nowhere even close to a million. And there's only 10 days left. So it looks like the million dollars is the best deal. Of course, we suspected that all along.

Fourth Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
22	20971.52	41943.03
23	41943.04	83886.07
24	83886.08	167772.15
25	167772.16	335544.31
26	335544.32	671088.63
27	671088.64	1342177.27
28	1342177.28	

Hold it! Look what has happened. What's going on here? This can't be right. This is amazing. Look how fast this pay is growing. Let's keep going. I can't wait to see what the total will be.

Last 2 Days – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
29	2684354.56	5368709.11
30	5368709.12	10737418.23

In 30 days, it increases from 1 penny to over 10 million dollars. That is absolutely amazing.
(\$10,737,418.23)

Questions to consider:

If your boss was so impressed with your reasoning skills that he kept you on for 10 more days and pay you using Payment Option 1. However, since your help is so costly, he is now only willing to give you 50% more each day after the 30th day.

1. How can you determine how much money he would receive on Day 35?

multiply by .5 and add 5 times OR multiply by 1.5 5 times

2. How can you determine how much money he would receive on Day 40?

multiply Day 30's pay by 1.5 ten times

3. If you know how much money he receives on a certain day, how can you determine how much money he will receive 2 days later? ... 10 days later?

2 days later \rightarrow starting amount $(1.5)^2$

10 days later \rightarrow starting amount $(1.5)^{10}$

4. Write a sentence that describes how much money the guy receives each day after day 30.

he earns 1.5 times as much as the previous day

5. What is the rate of change? Is this a common difference or a common ratio?

multiply by 1.5, common ratio

6. Use the words *NOW* and *NEXT* to write a rule to express the pattern.

$$\text{NEXT} = \text{NOW} \cdot 1.5$$

7. Use the pattern you discovered to write an explicit equation for the rice acquired each day. (number of day, \$ received)?

$$y = 5368704(1.5)^x$$

x = days after day 30
 y = money received

8. Graph the first ten days of salary option 1 on the graph to the right. Be sure to label your axes and title your graph. Is this a graph of a linear function or an exponential function?

exponential

