Half-life Problems

Recall: The half-life of a radioactive substance is the time it takes for half of the material to decay. You are encouraged to make a table in order to generate some of the data for each problem situation below. Solve the following half-life problems by writing an equation and using the equation to find the solution. Make sure you find the initial value for each equation. The first problem has been **partially worked** in order to help you with the remaining problems.

1) A hospital prepared a 100-mg supply of technetium-99m, which has a half-life of 6 hours. Use the table below to help you understand how much of technetium-99m is left at the end of each 6-hour interval for 36 hours. Use this to help write an exponential function to find the amount of technetium-99m that remains after 75 hours.

The amount of technetium-99m is reduced by one half each 6 hours as shown in the table below. Fill in the missing information in the table and in the equation below.

Number of 6-hour Intervals	0	1	2	3	4	5	6
Number of Hours Elapsed	0	6	12	18	24	30	36
Amount of Technetium-99m (mg)	100	50	25	12.5	6.25	3.13	1.56

The amount of technetium-99m is an exponential function of the number of half-lives. The initial amount is 100 mg. The decay factor is 1/2. One half-life equals 6 hours.

Write an explicit equation if x = the number of 6-hour intervals. Y = $100(1/2)^x$

That's getting really easy to do now! But...what if x = the number of hours elapsed. It would be easier to plug in the number of hours instead of how many "6-hour intervals", but we would have to change the equation a little.

If x = the number of hours elapsed, then the number of 6- hour intervals (of half-lives) = $\frac{1}{6}x = \frac{x}{6}$.

Equation: $y = 100 \left(\frac{1}{2}\right)^{\frac{x}{6}}$

Use your equation to find the solution to the question.

$$y = 100 \left(\frac{1}{2}\right)^{\frac{x}{6}}$$

$y = 100\left(\frac{1}{2}\right)^{\frac{75}{6}}$ → HINT: When you use rational exponents in your calculator, put ( ) around them!

$y = 0.017$

After 75 hours, about .017 mg of technetium-99 remains.

Use a similar format in order to find the equations and solutions of the 4 remaining problems.

2) Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-gram sample. Use the function to find the amount remaining after 6 days. (Hint: Make a table to help you understand the data.)

$$y = 90\left(\frac{1}{2}\right)^{\frac{6}{17.5}} = 70.96\text{ g}$$

3) Phospohorus-32 is used to study a plant's use of fertilizer. It has a half-life of 14.3 days. Write the exponential decay function of a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

$$y = 50\left(\frac{1}{2}\right)^{\frac{84}{14.3}} = 0.85\text{mg}$$

4) Iodine-131 is used to find leaks in water pipes. It has a half-life of 8.14 days. Write the exponential decay function for a 200-mg sample. Find the amount of iodine-131 remaining after 72 days.

$$y = 200\left(\frac{1}{2}\right)^{\frac{72}{8.14}} = 0.43\text{ mg}$$

5) Some radioactive ore which weighed 20 grams 200 years ago has been reduced to 12 grams today.

a. Use exponential regression on your calculator to write an exponential decay function in order to find the solution. $y = 20(0.997)^x$

b. Based on your equation, what is the half-life of this radioactive ore?

$y_1 = 20(0.997)^x$ point of intersection
$y_2 = 10$ shows that half-life is 271.38 years.

c. Based on your half-life, write another exponential equation for the data in which the base of the exponent is ½ .

$y = 20\left(\frac{1}{2}\right)^{x/271.38}$

d. How much will be left in 400 years?

7.2 grams (same answer in either equation)

Adapted from Prentice-Hall Mathematics Algebra 2, Pearson Education, Inc., Upper Saddle River, NJ, 2004.

More Half-Life Problems

Most things are composed of stable atoms. However, the atoms in radioactive substances are unstable and the break down in a process called radioactive decay. The rate of decay varies from substance to substance. The term **half-life** refers to the time it takes for half of the atoms in a radioactive substance to decay. For example, the half-life of carbon-11 is 20 minutes. This means that 2,000 carbon-11 atoms will be reduced to 1,000 carbon-11 atoms in 20 minutes, and to 500 carbon-11 atoms in 40 minutes.

Half-lives vary from a fraction of a second to billions of years. For example, the half-life of polonium-214 is 0.00016 seconds. The half-life of rubidium-87 is 49 billion years.

In the problems below, write an exponential decay function in order to find the solution to each problem. (Use function notation)

1) Hg-197 is used in kidney scans and it has a half-life of 64.128 hours. Write the exponential decay function for a 12-mg sample. Find the amount remaining after 72 hours.

   $y = 12\left(\frac{1}{2}\right)^{\frac{x}{64.128}}$ in 72 hours there will be 5.51 mg

2) Sr-85 is used in bone scans and is has a half-life of 64.9 days. Write the exponential decay function for an 8-mg sample. Find the amount remaining after 100 days.

   $y = 8\left(\frac{1}{2}\right)^{\frac{x}{64.9}}$ in 100 days there will be 2.75 mg

3) I-123 is used in thyroid scans and has a half-life of 13.2 hours. Write the exponential decay function for an 45-mg sample. Find the amount remaining after 5 hours.

   $y = 45\left(\frac{1}{2}\right)^{\frac{x}{13.2}}$ in 5 hours there will be 34.61 mg

4) Carbon-14 is used to determine the age of artifacts in carbon dating. It has a half-life of 5730 years. Write the exponential decay function for a 24-mg sample. Find the amount of carbon-14 remaining after 30 millennia (1 millennium – 1000 years).

   $y = 24\left(\frac{1}{2}\right)^{\frac{x}{5730}}$ in 30 millennia there will be 0.64 mg

5) A decaying radioactive ore originally weighs 27 grams and is reduced to 18 grams in 1,000 years.

   a. Use exponential regression on your calculator to write an exponential decay function in order to find the solution.

      $y = 27(0.99959)^x$

   b. Based on your equation, what is the half-life of this radioactive ore?

      $y_1 = 27(0.99959)^x$
      $y_2 = 13.5$
      point of intersection shows the half-life is 1709.5 years

   c. Based on your half-life, write another exponential equation for the data in which the base of the exponent is ½ .

      $y = 27\left(\frac{1}{2}\right)^{\frac{x}{1709.5}}$

   d. How much will be left in 3,000 years?

      8 grams

Half-life: Determining and Graphing the Half-life of M&Mium

Background: You should know the term "half-life" and know how it is related to radioactive elements. The half-life of a radioactive element is the time it takes for half of its atoms to decay into something else. For example, iodine-125 (I-125) has a half-life of about 60 days; therefore, in 60 days, 1g of I-125 will turn into half a gram of iodine-125 and half a gram of something else (the radioactive decay products of radium). After another 60 days have elapsed, only a ¼ of the original 1g of I-125 will remain.

Purpose: To determine the half-life of the element M&Mium.

Materials:

- Bag of M&Mium Isotopes
    - Radioactive members of this isotope family are distinguished via a bold **m** on the front surface of the atom.**

- 1 plastic cup
- pencil/pen
- white piece of paper
- 1 sheet of graph paper

Procedure:

1. Count the number of M&Mium atoms as you place them in the cup. Record the total number of radioactive atoms you start with in your data table.
2. Cover and shake/rattle the cup.
3. Carefully pour your atoms onto your white paper. You will see that several of the previously radioactive atoms in the group have "decayed", and the **m** is no longer visible. Set these atoms aside. Count how many M&Mium atoms you have remaining (the ones with **m** still facing up) and put that new total in your table.
4. Put the remaining atoms back in the cup & continue the pattern until no more radioactive M&Mium atoms remain. Remember to record the number of radioactive atoms left after each shake!
5. Using the graph paper provided, plot the data from your table.

Answers will vary!

Number of Shakes	Remaining M&Mium atoms
0	43
1	22
2	9
3	5
4	3
5	2
6	1
7	1
8	0
9	
10	

Conclusions:

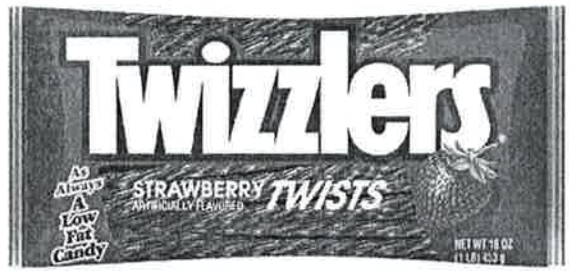
1. Calculate the half-life of M&Mium? (i.e., What number of shakes are necessary to reduce the radioactive members to one-half?) approximately 1
2. What does the NOW-NEXT equation look like?
   NEXT = NOW (½), start at 43
3. Compare your NOW-NEXT equation to those of a partner. What are the similarities and differences?
   different starting numbers
4. What does the explicit equation look like in function notation?
   $f(x) = 43(\frac{1}{2})^x$
5. Compare your explicit equation to those of a partner. What are the similarities and differences?
   different starting values
6. If you had 1000 M&Mium atoms,
    a. How many atoms will remain after 5 shakes?
       about 31
    b. How many shakes would it take before there are no more radioactive atoms remaining?
       around 11 shakes (0.488 M&Mium atoms rounds to 0)

Half-life: Determining and Graphing the Half-life of a Twizzler

Background: You should know the term "half-life" and know how it is related to radioactive elements. The half-life of a radioactive element is the time it takes for half of its atoms to decay into something else. For example, iodine-125 (I-125) has a half-life of about 60 days; therefore, in 60 days, 1g of I-125 will turn into half a gram of iodine-125 and half a gram of something else (the radioactive decay products of radium). After another 60 days have elapsed, only a ¼ of the original 1g of I-125 will remain.

Purpose: To determine the half-life of a Twizzler and graph the results.

Materials:
2 Twizzlers (1 for Part I and 1 for Part II)
Plastic knife
pencil/pen
2 sheets of graph paper



Procedure: Part I: Amount of Twizzler vs. Bites

1. Hold original Twizzler vertically against the 'y' axis with one end at the origin. Mark the "length" on the y-axis. This represents the beginning amount.
2. Wait for further instructions to "Take a ½ bite!" You must eat HALF (and *only* half) the length of the Twizzler. (Or use a plastic knife to cut the twizzler in half).
3. Move the remaining Twizzler to the one unit right on the x-axis. Mark the new length (this is your *y*-coordinate).
4. Repeat steps 2 and 3 with the class until the instructor tells you to stop.
5. Draw a smooth curve through the points you graphed.
6. Make a table of your data from steps 1-4 below.

Number of Bites	0	1	2	3	4	5
Amount of Twizzler (cm)	20	10	5	2.5	1.25	.625

Answers will vary!

7. What is the initial value? What does it represent in this situation?
   20 cm represents the original length of Twizzler
8. Do the dependent values in your table represent an arithmetic or geometric sequence? Determine the common ratio or difference based on your answer.
   This is a geometric sequence and the common ratio is $1/2$.
9. Write a NOW-NEXT equation for the situation.
   NEXT = NOW($1/2$), start at 20
10. Write an explicit function for the situation in both "y=" and "f(x)" form.
    $y = 20(1/2)^x$ $f(x) = 20(1/2)^x$
11. Use the f(x) equation to find out the length for 10 bites or f(10).
    $f(10) = 20(1/2)^{10} = 0.0195$ cm
11. If you keep halving the Twizzler will it ever completely disappear? Explain your thinking.
    Mathematically, no, half of something will never be zero, but physically, you can't bite something very precisely when it gets that small.

Procedure: Part II: Amount of Twizzler vs. Time

Let's say it takes you exactly 45 seconds to eat (or cut off) half the Twizzler. Fill in the table based on your values in Part 1.

Time (seconds)	0	45	90	135	180	225	360*	720*
Amount of Twizzler (cm)	20	10	5	2.5	1.25	.625	.078	.00031

*CAREFUL!!!!

12. In this scenario, what is the half-life of the Twizzler?

45 seconds

13. Write a NOW-NEXT equation for the situation.

NEXT = NOW(1/2), start at 20

14. How many times will the Twizzler have been halved after each of the following amounts of seconds?

a. 0 — 0
b. 45 — 1
c. 90 — 2
d. 135 — 3
e. 180 — 4
f. 225 — 5
g. 360 — 8
h. 720 — 16

15. How did you determine the answers to number 13? Compare your method to those of your classmates.

Divide the number of seconds by 45.

16. Write an explicit ("y=") equation for this new situation. Let x = the number of seconds. Plug in the x-values from your table to make sure that your explicit equation really works. If it doesn't, make changes until it does.

$y = 20\left(\frac{1}{2}\right)^{\frac{x}{45}}$

17. Use the f(x) equation to find out the length after 405 seconds.

0.039 cm

Extension:

1. If you had started with a GIANT Twizzler (2X the normal size), how would this have affected the shape of the graph? Explain.

The shape would be the same, but the graph would have a y-intercept of 40 instead of 20.

2. Write an explicit equation if you took a bite every 90 seconds. Let x = the number of seconds.

$y = 20\left(\frac{1}{2}\right)^{\frac{x}{90}}$